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Information was exchanged on the psychological and sociological problems involved in teaching mathematics to underachieving students. Areas of concern were: great city slum schools, all-Negro schools, and mathematics for students of low ability. A project to increase literacy in these students was recommended.

The conference report was published under separate cover as an appendix to the School Mathematics Study Group (SMSG) Final Report, and was intended to suggest specific supplemental research.

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CONFERENCE ON MATHEMATICS EDUCATION FOR BELOW AVERAGE ACHIEVERS

April, 1964

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CONFERENCE ON MATHEMATICS EDUCATION FOR BELOW AVERAGE ACHIEVERS

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PREFACE

This conference resulted from a request by the Advisory Board of the School Mathematics Study Group that information be gathered on students whose achievement in mathematics is below average, for any one of a variety of reasons, and that recommendations be made for both experimentation and curriculum development appropriate for SMSG to undertake.

Through the interest of the Cooperative Research Branch of the U. S. Office of Education, funds were made available by the Office of Education for an exploratory conference. This conference was held in Chicago, Illinois, on April 10 and 11, 1964.

This report of the conference contains the seven papers prepared for the conference, together with summaries of the discussions of these papers by the conference participants.

In addition to the prepared papers, a panel of four mathematicians was asked to react to the papers and to the discussions. Transcripts of the remarks made by the members of the panel are included in this report.

Further, Mr. Frank Allen, president of the National Council of Teachers of Mathematics, presented a summary report of a similar conference* sponsored by the U. S. Office of Education in cooperation with the National Council.

Finally, this report contains suggestions and recommendations for future action which were made by the conference groups.

During the second day of the conference, the participants met in four groups, each devoted to a particular area of concern. The recommendations of the four groups were presented and discussed at a final plenary session.

These draft recommendations were then circulated to all the participants with a request for comments and clarification. The recommendations which appear at the end of this report reflect the suggestions which were received.

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* A preliminary report of this conference is available from the National Council of Teachers of Mathematics, 1201 16th St., N. W., Washington, D. C.

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E. G. Begle

PURPOSE OF THE CONFERENCE

In the past decade an enormous amount of work has been done to improve the mathematical programs for the schools in this country. A large number of projects have been at work. The School Mathematics Study Group, which has called this Conference, is only one of these projects.

All these projects have behaved in a very sensible fashion: they started with the easier problems. This was sensible because it meant that some progress could be made very quickly and some experience could be gained that would be helpful in looking later at the more difficult problems.

The easier problems were those of the high school programs for average and above average students, the junior high school programs, and, finally, the elementary school programs.

From the very beginning SMSG recognized perfectly well that we were doing something for only part of the school population. We have made a remarkable amount of progress, but we are now far enough along to realize that the rest of the school population, the students who are not doing well in mathematics, must be given attention.

Once we recognized that it was time to face up to this problem, it did not take long to see that we have a whole spectrum of problems to consider. Those who do not do well in mathematics do not do well for a great variety of reasons. Most of these we have not had to be concerned with so far, and most of us are not well acquainted with all of these problems.

A major purpose of this conference, therefore, is to try to point out to a representative group of individuals who have worked on mathematics curriculum projects what some of these new problems are like. The first day of this Conference, therefore, will be devoted to hearing about the general nature of the problems. Then, with this background information, we shall spend the following day considering explicitly what the School Mathematics Study Group should do next.

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Harry Beilin and Lassar G. Gotkin

PSYCHOLOGICAL ISSUES IN THE DEVELOPMENT OF MATHEMATICS CURRICULA FOR SOCIALLY DISADVANTAGED CHILDREN

In characterizing the learning problems of socially disadvantaged youth some well-meaning critics suggest that all persons are in some way disadvantaged either by lack of intelligence, social status or limitation in personality. These critics maintain that to respond to one group's disadvantage and not those of all groups represents a policy which is unfair and by implication educationally unsound. Although there may be something to recommend this view, it fails to recognize crucial distinctions among the 'disadvantaged'. Whereas the white middle-class child may differ from his fellows in intelligence or wealth, he is not without sufficient personal and social resources to overcome his particular 'disadvantage'. This luxury is not afforded the lower-class youth who very early in life loses the opportunity to develop the means which, at the minimum, would enable him to adapt to the economic and social life of the community. The socially disadvantaged children of whom we speak do not grow in an open economic or social system, where their achievements are in direct proportion to their efforts as much as some would like to think they are. The most unfortunate feature of their experience is that by the time they reach adolescence they are so limited in intellectual and social skills as to make a personal reversal in their fate extremely difficult. It, therefore, behooves all of us to strain our resources to provide such youngsters with the skills which will make an economically and personally gratifying life possible not only because of a moral imperative but for the enlightened self interest of the entire community.

CLASS DIFFERENCES IN ACHIEVEMENT

The achievement and intellectual differences between urban poor and advantaged children are no myth. By the time the average slum child reaches the seventh or eighth grade he is approximately two to three years behind his middle-class counterpart in reading skill. In one New York City junior high school located in a slum area the average reading score for one eighth grade class was 6.7; another eighth grade class in the same school had an average reading score somewhat above the third grade level (3.4). A number of children in these classes were not even able to read at the first grade level. The assessed intellectual performance of slum children is likely to be at least 10 to 15 IQ points lower on the average from those of other children, and scholastic achievement is likely to be retarded in all areas of academic learning. (The average IQ for the former eighth grade class cited was 89.4; for the latter 70.6). Interestingly enough, those areas of intellectual performance which show the least difference in level of performance are associated with mathematical reasoning (Rosner, 1956).[1] Factor analysis of intelligence test performance, on the other hand, indicates that the reasoning processes employed by both middle-class and lower-class children are the same. The social class differences in intelligence and achievement among seventh graders are generally greater than those of children who first arrive at school but the differences even at the early grades are striking. What is particularly distressing, however, is that the differences in the early grades are in "tool" skills, that is, in the skills that enable the child to process formal information offered in the school curriculum and informal information provided by his out of school experience. The urban slum child, whether Negro, Puerto Rican, Mexican, Indian or White has lived five or six years in an environment which both physically and socially has placed severe constraints upon certain aspects of his development. This is not true for all areas of his development, for the rigors of this existence force certain kinds of social adaptation upon the child that makes him more resilient and vigorous than some of the hot house varieties bred in suburbia.

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[1] This may be an artifact of the more restricted range of performance in mathematical reasoning than in other intellectual areas.

CONSTRAINTS IMPOSED BY SLUM EXPERIENCE - LANGUAGE DEFICITS

On the other hand, the lower-class home is limited in that the actual intellectual and social interchange between parents and children is of a kind that does not provide a practical and easy transfer to the school setting. In the lower-class home meal times, for example, do not represent a social experience in which there are opportunities to acquire and test one's language skills. Reading periods are practically non-existent. The most significant outcome of this is the very limited language facility of the slum child (Milner, 1951). Not only is there limited content in the language, but lower-class life both in the home and in the community fosters the development of a linguistic 'code' which is different from that of members of the middle-class. One British investigator (Bernstein, 1961, 1962) characterizes linquistic codes as 'restrictive' and 'elaborative'. He observes that lower-class language follows a restricted code pattern while middle-class environments foster the development of both elaborative as well as restrictive codes. The restrictive codes of the middle-class are different, however, from those of the lower-class. The middle-class restrictive code is usually of a special or formal language (e.g. mathematics), whereas the restrictive code of the lower-class might be, for example, the 'hip' language of some lower-class adolescents. The importance of these different orientations to language is that they are not equally suited to school instruction. Further, the acquisition of a limited linguistic code provides the child with a language structure which militates against the development of an extensive vocabulary and keeps thought at a low level of conceptualization.

Even if lower-class parents desire to play an active role in the education of their children their own lack of knowledge is so great as to make their functions as teachers very limited. On the whole, lower-class parents are far less effective as teachers than middle-class parents. Many lower-class parents act as though they subscribe to the view that if a child can't give the correct answer to a problem the best way to get it out of him is to threaten or physically coerce it out of him.

The lower-class parent is not only far less effective as a directive agent for learning, but is unable to provide the child with an environment adequate for learning. Books are not present and read in the home; neither is the home a very safe place to bring books. Children may be kept up very late by a blaring television set. They cannot be expected to have a place to study. The relative language isolation of children in these homes is aggravated by missing fathers, frequent geographic dislocation, poor diets, crowded space, and a high incidence of disease.

OTHER COGNITIVE DEFICITS

Limited language resources are not the only legacy of this experience. The lower-class child arrives at school with a wide variety of cognitive deficits. He is less capable for example of making perceptual discriminations among physical objects in his environment, less able to deal with the pictorial representation of objects and actions and more limited in the ability to conceptualize, in even primitive ways. In one of our studies with kindergarten children there were consistent social class differences in the judgment of equal and unequal areas, as well as in the ability to equate areas when their configurations were altered (Beilin, 1964). The tasks given these children were designed to determine whether: 1. two squares (subdivided into four units each) were equal in area, 2. a square (of 4 units) was equal or unequal to a figure with 5 or 3 units, and 3. a square was equal in area to other figures (also with 4 units) which were rearranged so that the patterns were not square. In another study we also found differences between lower and middle-class second and third grade children in the ability to imagine what the water level would be in line drawings of jars tilted in various orientations to the horizontal (Beilin and Kagan, in prep.). Almy has likewise found consistent differences between middle and lower-class New York children in the logical capacities to deal with the 'conservation' of number and volume problems described by Piaget. In these problems, for example, the child is asked to count the number of blocks he sees lined in a row before him. The same blocks are then spread out (in the same line). The child paradoxically insists there are now more blocks, even if he recounts them and recognizes they are the same number. (Almy and Chittenden, 1963)

NON-COGNITIVE PROBLEMS

The urban slum child who arrives at school with these limited cognitive resources is bound to find his experience very trying unless the school is prepared to recognize and specifically overcome these deficits, as well as those of motivation and interest. It would be naïve to assume difficulties reside solely in the cognitive and intellectual realms. It must be recognized by those who prepare curricula for disadvantaged children that they arrive in the classroom with other characteristics which inhibit learning. The disordered life in which the slum child grows does not foster the development of those internalized controls that keep a child attending to a task for protracted periods of time or enable him to attend to what the teacher is saying. These children are much more easily distracted than their peers and much more likely to act out their wants in physical ways. Requests are not mediated through language but through actions, which are often perceived as hostile and aggressive, and which tend to disrupt a classroom. Such children characteristically nudge or touch the teacher to get her attention. They take the ruler another child is using rather than verbally request it. The ruler is just as likely to be used as an extension of the arm in a fencing match as an extension of the arm for measurement. The order and self discipline required of the learner in school settings is alien to the experience of many of the children we are describing. This kind of self discipline is as necessary to learning as cognitive skills since teaching may be delayed until some semblance of personal order is achieved.

One is likely to find, too, that materials, devices and language which are new and different hold little attraction for them. They may be intrigued and stimulated by the initial novelty of the school setting but interest may progressively wane, as with all children, unless something more sustaining than novelty is introduced. If the novelty is not followed by some success in dealing with the newly encountered materials, in all likelihood, the frustrations resulting from unsuccessful attempts to master them will result in a retreat from the potentially interesting school environment.

LEARNING AND DEVELOPMENTAL PROCESSES

Mentioning these problems to mathematicians may appear in questionable taste. Mathematicians may feel theirs should be the problems of mathematics, while the context and conditions in which mathematical ideas and skills are taught are the responsibility of school administrators and teachers. Unquestionably, such problems are primarily the responsibility of school personnel and ordinarily far less of the mathematician's attention as curriculum developer should be directed toward them. However, our target populations are not ordinary. Recognition and acceptance of these problems may, in fact, prove crucial to both development and implementation of curricular programs.

If we are to anticipate what is possible in reversing the effects of the slum child's experience, we must examine the learning and developmental processes which have brought him to the point of being seen not only as socially, but also as intellectually disadvantaged.

To recognize that a child's ability to respond and learn may be causally related to his prior experience does not preclude the possibility of invariance in the order of his intellectual development. The work of Piaget on the development of cognitive functions in children demonstrates that such invariance exists (Flavell, 1963). He offers evidence for a series of developmental stages in which the intellectual and cognitive achievements of a particular period are dependent upon those of prior periods. His data suggest that if the experience of a child is limited both in character and extention only limited intellectual resources will result from the experience of a particular era in the child's life. The later intellectual development will thus be restricted since these subsequent developments will be dependent upon them. Although the sequence in cognitive development may remain unaltered by experience, what Piaget and others have observed is that the transitions from one stage of intellectual development to the next are influenced by various kinds of cultural and planned experience.

The earliest of the developments that may be so affected is the organization of perceptual-motor skills. The child's first responses to sensory experience are motor. In the crib, for example, the child responds to rattles and toys by accidentally hitting them and as a by-product discovering that they produce interesting sounds. He tends to repeat these actions and the coordinations between what he sees and how he acts become firmly embodied in his muscle and neural systems. Through what are at first unintentional actions he learns to coordinate his activity with his sensations, quickly acquiring a repertoire that makes later intentional action possible. It is through these primitive sensory-motor 'schema' that the child is first able to construct his reality. From direct action the child is gradually able to transform his experience to 'images' of action. Whereas the child at first has need to see a toy to know that it exists he gradually comes to the recognition that it exists even when not seen, as when hidden behind a screen or placed under a pillow. The manipulation of this imagery permits vicarious problem solving and elementary conceptualization. Such a child will now be able to obtain a marble from within a match box, testing first by putting his finger in his mouth, then pressing it against the side of the inner box. This is the beginning of symbolic representation - at this stage through action - in later stages through language and pictorial representation. The ability to deal with one's experience through the imaginal representation of action permits the child much greater scope in dealing with reality and provides the basis for more intelligent behavior. The acquisition of language permits even greater extension of the child's intellectual capacities, for it provides him not only with content for his thought but with symbolic tools. Although the young child (prior to age six) is endowed with language and imagery these have not fully liberated his thought from the service of his perceptions. As Piaget has clearly illustrated (Wohlwill, 1962) the young child's response to reality is largely in terms of what he perceives - principally by the way things appear. His conception of the number of objects before him becomes greater if they are spread out, water volume increases if spread thin, areas are made larger if they are partitioned and rearranged. By school age, however, the normal child develops a cognitive system that enables him, in spite of perceptual incongruities and ambiguities, to attain a conception of the invariance of 'physical' concepts (area, number, volume). In these problems (which Piaget says reflects 'conservation' abilities) the child is able to reason that the number of objects before him remain the same even if placed in different locations, and volumes remain invariant irrespective of the shapes of the vessels into which they are poured. In this era of the child's development, which lasts till about the age of 10, for normal children, the capacities for classificatory and relational thinking are developed. These intellectual abilities are limited, however, by the dependence of the child's thought upon the manipulation of concrete objects and their pictorial or schematic representations. It is not till early adolescence that the child's thought is liberated from the manipulation of concrete materials, and is able to deal with elements and relations in a wholly abstract and symbolic fashion. He becomes capable, in fact, of thinking in terms of formal propositions.

To put the Piaget developmental scheme more succinctly there are three general developmental periods in the child's development of cognition: sensory-motor, concrete operations and formal operations. The sequence of development is fairly fixed. Although the sequence is fixed, the age at which a particular child makes the transition from one stage to the next is not fixed. The variability in making these transitions is, in fact, considerable.

Evidence is available to indicate that the performance of urban slum children, at least in one stage in their cognitive development, is markedly inferior to that of middle-class children (Almy and Chittenden, 1963). In the stage of concrete operations when the 'conservation' capacities of children normally reach fruition Almy found about a 25% difference between the ability of slum children and advantaged children to perform correctly on 3 conservation tasks (in the first and second grades).

READINESS

A momentary diversion may be in order here. In speaking of stages, as we have, and the ability and inability to function at particular periods in the child's life it may seem that we are suggesting certain guidelines for when a child is 'ready' to learn or not to learn. This notion of 'readiness' has been considerably abused in recent times. Many people have been justifiably exercised over the idea that children have been kept from learning how to read till they were six because they were not 'ready'. It is very true that in educational circles this notion has for years had the force of law. It has been made clear of late, however, that as a generalization this makes little sense. In turn, the notion of readiness itself has been vigorously attacked. This is a good instance of the baby being thrown out with the bath water. What should have been clearly shown was that the evidence for the 6 year readiness rule was flimsy at best. It was the scientific evidence for this generalization which was at fault, not the general evidence with regard to 'readiness'. The readiness notion can be approached more fruitfully in a rather different way. Even some investigators who are convinced of the idea that children can be taught to learn most any concept if it is prepared for their stage of development place at least implied limitation on this generalization. Bruner, for example, cites a study (by Frank) in which children at two ages (4 and 5) are provided with verbal experience in solving a 'conservation' problem (Bruner, 1964). The older children are able to use the verbal information; the younger not. In another study it was likewise shown that it was not possible (albeit in a limited demonstration) to teach qualitative area measurement to first graders, but it was possible to teach qualitative length measurement to them (Beilin and Franklin, 1962). There _are_ limitations upon when and how children can learn. Our task is to discover what these limitations are and under what conditions they operate and with whom. The danger is in calling a halt too soon to efforts to discover what is possible in the child's learning. Nevertheless, it is just as much folly to say all children are prepared to learn at a particular age as it is to say no children are prepared to learn.

CONGNITIVE REPRESENTATION

In an attempt to view cognitive development from a somewhat different orientation, Bruner places greater emphasis on the processing of the data of experience. He suggests three ways in which experience is represented in the child's thought. The first type of representation is _enactive_, i.e., it results from motor activity. The experience of learning to ride a bicycle gets represented kinesthetically and muscularly. No amount of verbalization will help another learn this skill. The second type of representation is denoted as _iconic_: these are images of what is visually, tactually or auditorily perceived. The final type of representation is _symbolic_. Here the events of experience are represented in either natural or formal languages. The nature of development is such that one style of dealing with the world is successively supplemented by more encompassing modes of representation. The scope of the child's ability to act upon the world, as well as to be acted upon, is considerably enhanced by the more abstract and symbolic types of representation, but these are the most difficult of achievement.

The work of Krutetski is illuminating here (Krutetski, 1963). In a study of Russian children identified as having an "incapacity for mathematics" Krutetski identified two general components of intellectual activity as basic to mathematics learning: a 'visual-image' component and a 'verbal-logical' component. In testing an eightfold typology involving the interrelation of these two factors he identifies four groups of children who suffer from this incapacity for mathematics. What these groups of children have in common is the inability to solve mathematics problems posed in a verbal-logical form. (For example, "there are two circles with a radius of 2 cm. and 3 cm. The distance between their centers is 10 cm. Do those circles intersect?") The ability to deal with visual-image representations of the problems is not a consistent failing, i.e., these are children who have the visual-image capacities but who fail with the verbal-logical representations, and do badly in general with mathematics learning. Unfortunately little is reported of the 'backgrounds' of these poor learners so that the origins of these verbal-logical deficits is not clear. What does emerge from the Krutetski, Piaget and other studies is that there is a pronounced relationship between the ability to perform well in mathematics and the ability to deal with the symbolic and abstract representations of problems, whether verbally or nonverbally. Socially disadvantaged children are particularly deficient in the ability to represent their experience symbolically and this circumscribes their potentialities for learning mathematics.

From the viewpoint of both Piaget and Bruner, the crucial issue in cognitive development is the transition from one stage of development to the next - or from one form of cognitive representation to another. Although it has been possible for psychologists to identify and characterize the stages in cognitive development, it has not been possible to explain very satisfactorily how the transitions from one stage to the next are made. The mechanisms of change are rather obscure. This makes the task of the educator particularly difficult if he desires to accelerate the transitions when a child is retarded or slow.

The lower class child's intellectual performance is typically that of a less mature learner. The predominant pattern of thought and the types of materials that can be handled are those of younger middle-class children. The slum child, upon arrival at school, is more characteristically an 'enactive' child than the middle-class child. He is also less able to deal with concrete materials, as well as the pictorial or schematic representations of them. For example, when given blocks to sort according to particular characteristics (e.g., size, color, form) or when given pictures of objects or line drawing representations of them, the slum child will have greater difficulty recognizing the objects or operating on them. In essence, he will have more difficulty with concepts related to the objects and with their symbolic representation than with the objects themselves. The disparities between children becomes greater as the middle-class child acquires greater competence in the symbolic realm, while the lower-class youngster continues to struggle with the concrete materials.

TRANSITIONS AND TRANSLATIONS

The principal task for work with disadvantaged children as we see it, is to facilitate the transition from one level of thinking to the next through the development of special instructional techniques and materials. Children who are particularly motoric or 'enactive' will profit from the translation of this kind of activity to reasoning with perceptual (visual, tactual) types of representation. Those who demonstrate facility at the perceptual level, in turn, require a translation to the symbolic level, particularly with the special language with which we are here concerned, the language of mathematics. For example, a child who is able to sort out large from small blocks may be able to learn to do the same with the pictures of blocks, or houses. He may then be taught to associate the words 'large' and 'small' with each of the groups. The child may progressively move to the point of being able to deal with the logical relationship between large and small verbally (symbolically) without direct reference to the physical objects. The task can be made more difficult by having the child deal with three blocks of different size or three houses of different size - and come to recognize that the middle one is both smaller than the larger and larger than the smaller, and becomes able to verbalize it. He may be trained to carry the understanding of this relationship to other objects, and then to other kinds of relations such as 'long' and 'short', 'wide'-'narrow',etc. The development of translational techniques (i.e. of learning the logical equivalence of a concept represented visually and verbally) requires a prerequisite knowledge of the methods of instruction with each type of representation. The most difficult of the skills, however, are the translational skills themselves by which it is indicated how reasoning in one mode is equivalent to reasoning in another. These techniques are not altogether new to mathematics instruction. Some materials already do this. The Cuisenaire rods, for example, do this. From a deliberate use of narrow lengths of wood divided by color into equal units one moves to symbolically representing and manipulating the numerical relations among the 'rods'. Many of the Montessori materials do the same.

An illustration of the Montessori approach is seen with the 'counting beads'. When ten individual beads have been obtained from a receptacle by the child they are exchanged for a string of ten beads. Ten such strings may then be put together in a square frame. Ten of these squares can be piled up and shown to be equivalent to a cube of 1000 beads. In this manner children physically handle sets of ten, sets of 100 and sets of 1000 with the units observable within the sets.

Montessori methods have been built, in large measure, on the notion of translation from sensory experience to the symbolic. It should be recalled that Montessori did not begin her work with the middle-class children who typically attend Montessori type schools now, but with "street urchins" in the San Lorenzo slum section of Rome. The basic techniques for which she became known were developed with them. As instructors who use these and similar materials recognize, once facility is developed in utilizing the formal language system, the manipulation of the concrete materials may and should be set aside.

The use of concrete materials to provide a basis for learning abstract relations is not only appropriate at the level of learning 'larger and smaller' and computation in a 10 based number system. The approach may be applied at more advanced levels, as in the use of a balance in learning quadratic equations. Dienes' work is designed, in part, to establish isomorphisms between the logical organizations among concrete materials with the same relationships represented symbolically and may provide a useful model in developing an approach to educating disadvantaged children (Dienes, 1963).

The most important of the translations that are to be made are those that bring the child from reasoning 'concretely' to reasoning symbolically. But even thinking in association with the concrete materials (as in numerical relations among blocks, or the beads of an abacus) may require a great deal of training for disadvantaged children before the translation can be made into natural language equivalents and to formal notation. It should be noted that much of the success of the so-called "new" mathematics has been demonstrated with middle-class children. The ability of young children to utilize sophisticated notational systems has not been demonstrated with disadvantaged children.

COGNITIVE EFFICIENCY

Neither the child nor adult functions wholly at the highest level of intellectual performance of which he is capable. He employs the kind of thought process most efficient to the demands of the problem and this may or may not be the most sophisticated thought process available to him. In one of the aforementioned studies (Beilin, 1964) it was found that children utilized an iterative measurement method to establish the equality between 4 unit area patterns that were differently arranged, but used a translocational method (i.e. ideationally moving the units to other positions) when the patterns contained 9 units, that is, when the patterns contained too many units for the children to count easily. This suggests that two kinds of information should be sought for every cognitive learning experience: first, the kinds of logical methods that may be utilized by the learner to solve a problem, and second, the appropriate manner of presenting problems in order to elicit particular logical solution methods. In this way one will ensure that the child is able to master a maximum of solution strategies.

THE DISADVANTAGED AS SLOW LEARNERS

Some educators of disadvantaged youth with whom we have discussed these issues have characterized disadvantaged children as slow learners and advise that instruction should proceed with the techniques developed for slow learners. Although it is true that many disadvantaged children are slow learners this position tends to overlook some distinctions. One group of slow learners consists of those with limited genetic endowment, whom the best of instruction would profit in only limited ways. Disadvantaged children are not slow learners in the same sense. Their progress can be much more rapid with appropriate techniques of instruction. A study by Revoire (Jensen, 1963) demonstrates the differential capacities of two groups of children with low IQs, one white and one Mexican. The latter group profited from the introduction of verbal materials in their learning: the white lower IQ children did not. It was reasoned that these particular Mexican children represented a socially disadvantaged group whose performance provided only a limited view of their potential intellectual status. This suggests that under the proper circumstances the introduction of verbal techniques of instruction may foster rapid learning with some disadvantaged children, whereas they might not with children who are slow learners because of mental retardation.[2] To put it another way, the optimal techniques of translation for one group of slow learners may be different from those of another. Needless to say, very little has been done in experimenting with these techniques.

THE AGE OF INSTRUCTION

Since so many of the problems of learning with the socially disadvantaged start at a very early age it would be desirable to initiate programs related to mathematics instruction at the earliest school age possible; even in the kindergarten and nursery school. In preschool classes, pre-mathematics programs should emphasize perceptual training and concept development relative to space, number, time and related conceptual domains. These will later become the basis for the development of conceptual systems symbolically represented. The development of curricula should proceed from the early grades upward.

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[2] As was brought out in the discussion of this paper there are learners who may be characterized as 'slow' who are able to achieve the same levels of performance as 'normal' or 'fast' learners but need more time to do so. We are not referring here to this group but to those learners who need augmented or special instruction to achieve better. In these circumstances the term 'slow' may be a euphemism for 'retarded'.

REDUNDANCY AND THE 'EDUCATION OF ATTENTION'

We have indicated earlier that an important difference between the lower and middle-class child is the greater distractability and limited attention span of the former. It has been implied by some researchers that conceptual learning with young learners requires building into their training procedures a maximum of 'redundancy' (Wohlwill, 1962). Redundancy can take two forms. Either material is repeatedly presented with no new information added, or else the same concepts are repeated in new contexts or in new forms, where again little or no new information is added. According to the 'redundancy' view a greater encounter with the learning materials is required for young and slow learners to acquire a concept. All who have worked with disadvantaged children know this to be true for them. Older, brighter and advantaged children require less redundancy. There is a danger, however, in merely repeating the same materials ad nauseum in the hope a child will learn. It is the danger of boredom and frustration for both the pupil and teacher. Fashioning multiple examples of the same concept may provide the opportunity, however, for such learning. The type of approach suggested by Dienes and others (Churchill, 1961), in which a variety of instances for representing a concept is offered the child articulates with this notion. Dienes' intent is to provide the child with a variety of representations of the same concept in order that the child may learn to abstract the concept from its many instances, but it might be more parsimoniously argued that the many new representations of the concept merely provide the child with more opportunities to encounter and thereby process the information necessary to develop a concept (whether by a process of abstraction or by some other process).

Coupled with the need for 'repetition' is the need for the units of instruction to be smaller, since slow learners in general have difficulty in remembering or retaining information even very recently acquired. In the language of information theory these children have rather limited channel capacity and to overload them will only break down the communication system between teacher and learner.

Having presented this view of concept learning we wish to offer another which is presently receiving considerable attention among psychologists. It suggests that in perceptual and cognitive learning the crucial determinant in learning is the 'education of attention' (Gibson, 1963). The child learns to attend to the critically differentiating details, progressively eliminating a concern for the irrelevant. The objective of teaching would be the identification of the relevant dimensions of difference and similarity in the attainment of concepts. Redundancy is minimized as the relevant details became more salient. This approach is evidenced in the ways in which children may learn the alphabet. For the Institute of Developmental Studies we have devised an 'alphabet board' (Deutsch, Beilin, Pollack) made of a display panel with alphabet cutouts into which the child may fit wooden letters. Attempting to fit the letters into the board draws the child's attention to the details of form and contour. Preschool children become progressively more adept at discriminating and identifying the letters through this sensory-motor training. This kind of training has relevance for first grade learning to read since there are data to indicate that there are significant differences between middle and lower-class Negro children in knowledge of the alphabet upon entering the first grade (Weiner and Feldman, 1963).

CURRICULUM PHILOSOPHY

There have been two philosophies of mathematics instruction which have divided practitioners in their work with children. These views are being carried over to work with the disadvantaged. One group which has its origins in the Froebelian tradition places its emphasis upon the attainment of mathematical principles through varied experience. The child is presented with the opportunity to manipulate meaningful materials. In response to the problems encountered with these the child develops intellectual skills. At one end of the continuum, this practical experience may take the form of dealing with the problems of paying and receiving in a store (Churchill, 1961), or at the other end, with the manipulations of a balance (Dienes, 1963).

The extent of external manipulation of the learner's experience depends upon the orientation of the teacher, but the intent is for principles and generalizations to be abstracted or discovered from experience. Teachers of the disadvantaged who work in this tradition say they can utilize the language and other modes of response already available to the child, even if these do not fit into accustomed patterns. This approach, they hold, permits the development of greater intellectual fluency and has the advantage of utilizing the child's prior experience.

The alternative view is oriented to providing the learner with a logical structure or grammar which he may apply to a variety of physical elements and relations. Here the materials are developed in a logically ordered fashion so that the child acquires fundamental logical principles in a sequential order gradually developing more complex and sophisticated skills. At one end of the continuum, one may have the sequential programs of Montessori (Montessori, 1948) starting with sensory experience to, at the other end, the deductive strategies of those who derive theorems and provide the opportunity to apply them to varieties of problems. Teachers who work in this tradition hold that the disadvantaged child, who is lacking in most cases in cognitive structures or the symbolic means of representing his experience, profits from first acquiring an understanding of relevant logical relationships. He is thus enabled to bypass the arduous and frustrating task of attempting to discover principles which to middle-class children (with their considerably broader referential systems) come more easily.

At this stage in our knowledge of mathematical learning it is not possible to adjudicate the differences between these views. There is undoubtedly merit in each. A decision as to which strategy will result in greater returns in learning is an empirical question which will require much more research and demonstration.

EXPOSURE TO INSTRUCTION

The fact that disadvantaged children require much more instruction than can be provided by the regular school program has suggested the development of supplementary learning programs to the regular school curriculum. This has been attempted in New York, for example, in an effort to improve reading ability. To foster such improvement special tutorial after-school programs are set up to accelerate the achievement of interested children. Since the tendency in urban schools is for classes to be crowded with large numbers of children, it seems desirable, if not necessary, to develop materials that may be used by children as a supplement to their regular in-class instruction.

CURRICULUM SUGGESTIONS

Out of these many observations we come to the point of considering specific suggestions for mathematics curriculum development.

One curriculum specialist (Bristow, 1964), in reflecting on curriculum proposals for reducing school dropouts, cited a number of curriculum positions which have already been accepted in a variety of quarters: 1. Elimination of the practice of failing students as a matter of curriculum policy, 2. meeting individual differences (in classes, schools and interests), 3. rejection of homogeneous grouping, 4. elimination of tracking systems, 5. a refusal to accept logically arranged materials, and 6. a rejection of grade organization of the school. He noted in each instance where this had been done that there was no program that was adequately thought through or had been experimented with in anything resembling a systematic procedure. He noted further that, whether the alternatives were wise or not, curriculum development had resulted more from the 'insights' and experience of teachers and supervisors who have worked with the disadvantaged than upon research or conceptually based programs of curriculum development.

There are, then, no ready curriculum strategies available for working with disadvantaged children. What this adds up to is that a group like the School Mathematics Study Group has an immense opportunity and responsibility in addressing itself to the problem. The opportunity is that presently there is no competition; the responsibility, that a mediocre job might prove immensely successful in comparison to present practice.

While we have no major strategy to offer we do have some suggestions. In offering these suggestions we will assume that the major subject matter decisions will have been made, that is, that the goals of mathematics instruction in terms of concepts and skills to be learned will have been specified.

A pilot project of the Biological Sciences Curriculum Study (Grobman, 1964) similarly specified their goals in developing special materials for the "750,000 tenth-graders who either take a special slow-learner course in biology or who are assigned to a rather elementary type of special 'general science' course." The

BSCS pilot study has been concerned with testing the feasibility of an approach to teaching complex biological concepts to slow learners. Many of the classes which were designated as slow-learner classes, however, had aptitude levels considerably higher than that found in classes of socially disadvantaged children. Two important guidelines set for the special programs were, first, that the program should stress laboratory work and second, that the reading level and length of reading assignments should be appropriate to the reading level of the students. Even though the researchers evaluated the pilot work as encouraging and are convinced of the feasibility of such an approach, it should be pointed out that the average post-test achievement for one of the learning units (across all of the classes) was only 55%. While the results may indicate learning above that usually found in such classes it seems important to emphasize that with 55% success the instructional problem was far from solved.

Suggestion 1: Begin at the first grade. Our impression of the SMSG curriculum is that work started at the high school level and worked down the grades. While expediency and fortuitous circumstance may lead to entering the curriculum in the middle years of schooling, it is less likely that such an approach will lead to a coherent mathematics curriculum for disadvantaged children.

Mathematics instruction in the traditional sense should include skills and concepts usually assumed to have been learned prior to the first grade. One has to ensure that children can make the necessary perceptual discriminations (e.g. large-small, tall-short, etc.) upon which other physical concepts will be dependent. In turn these physical concepts will lead to the abstract relations which are independent of particular physical events or objects.

Suggestion 2: Subject the curriculum to a behavioral and cognitive analysis. The strategy we propose involves three steps: 1) conceptual analysis of the materials, i.e. analysis of the logical order of the materials; 2) analysis of the cognitive learning and response systems required of the learner and 3) empirical testing. These procedures require careful specification of what is to be taught and what behaviors and cognitive processes the learner is assumed to have in order to cope with the instruction sequences. This strategy requires the empirical testing of the adequacy of the analyses.

In testing instructional sequences with children from disadvantaged backgrounds, a number of researchers have discovered that they have assumed a variety of behaviors and capacities that were missing to the children. Many of these behaviors were ones that they did not even realize were required for the learning nor were many of them easy to identify or remedy (Gotkin and Goldstein, 1964). Resnick, (1963) for example, describes the procedures used in revising a programed instructional sequence on the grid coordinate system to make it suitable for culturally deprived children. After five revisions the program still could not be considered successful, since 40% of the target population did not learn. Attempting such an adaptation, however, demonstrated how this process could help identify some problems in teaching culturally deprived children.

We have found it valuable at the Institute for Developmental Studies to conduct the empirical testing with individual children. We have an experimental psychologist, a master teacher, where appropriate a subject specialist, and a research assistant present for such observations. The research assistant or the master teacher presents the material to the individual learner while the others observe. The value of this approach was demonstrated with some materials designed to teach children to perceive the differences between pairs of pictures representing things that are visually very similar but conceptually different (such as a rhinoceros and hippopotamus, a squirrel and a chipmunk, a desk and a table). At first the child could not respond to what seemed a simple question. But rewording the question produced strikingly different results. In group testing of learning sequences one or two children are usually able to offer the correct response and thereby mask the difficulties encountered by the other children.

Suggestion 3: Utilize or develop techniques which will ensure the orderly transition to symbolic representation.

This embodies our most important point with regard to working with disadvantaged children.

Take the learner where you find him but be prepared through the preparation of adequate materials and techniques to move as rapidly as possible to learning mathematics on a symbolic level.

The stress in the early materials should be on sensory-motor manipulation. Later materials can be pictorial and still later, materials can be abstract and symbolic. The presentation of materials may be either simultaneous or sequential depending on the learning status of the child.

Develop the techniques for making the isomorphism between the conceptual representation in visual form and those in the symbolic form evident to the child. Further, provide sufficient experience with the articulation of the two until the child is able to proceed with some assurance solely with the symbolic.

Sub-suggestions:
a) Limit the size of the instructional units.
b) Provide for a variety of ways of representing a concept.
c) Provide for different levels of difficulty within a conceptual scheme.

Suggestion 4: Develop materials so that the concepts represented provide a challenge for the child.

The problems should represent some cognitive uncertainty (Smedslund, 1964). It should be an uncertainty, however, that can be reduced by the utilization of learning previously acquired in the program. This is a departure from approaches which place great stress on the reinforcement of appropriately made responses. Although we see considerable value in some 'drill' procedures (particularly with mathematical 'facts') we feel the focus of the program should be on training that will maximize the child's active solution of problems and to some extent the self-generation of the strategies of solution. Again, this should not preclude providing verbal or visual models where it is felt these will give greater mileage to the child's problem solving.

Suggestion 5: Provide supplementary materials that are self-instructional.

We have emphasized that slum children live in homes that are not conducive to studying (i.e. getting their homework done) and that their schools are characteristically overcrowded. Also, the children are mobile: Reports of 100% turnover in a classroom in a semester are not uncommon. Teachers are less likely to be well-trained and are themselves more transient. It is necessary if one is concerned with developing effective curricula to take these limitations into consideration. It is necessary to construct curriculum materials that do not depend on the active participation of parents in the home. These materials must also be suitable for school settings that are minimally organized, and flexible enough for students who enter in the middle of the term to cope with.

For these reasons it is suggested that some supplementary materials be as self-instructional as possible. One may look to the technology of programed instruction for the means of developing these materials.

Suggestion 6: Recognize the inadequacies of teachers.

The fact is that up to 25 per cent of teaching time in schools serving lower-class children is provided by teachers with emergency substitute licenses (Sexton, 1961). Teaching time provided by emergency substitutes, however, is four times greater in these schools than in schools serving middle-class populations.

It is essential to recognize, of course, that unlicensed teachers may be very fine teachers and even well trained, yet not meet particular licensing requirements.

Table 1 - Number and Percentage of Vacancies for Authorized Positions in Mathematics by School Level, 1962

School Level	Number of Authorized Positions	Number of Teachers without permanent (Math) Licenses	Percentage of Teachers without permanent (Math) Licenses
Academic Hi. Sch.	722	286	40
Vocational Hi. Sch.	93	63	68
Junior High School	1,350	988	73
Elementary School	60	51	85

Source: New York City Board of Education Report (1962)

From Table 1 it is evident that in a major city like New York the number of teachers not licensed to teach mathematics and yet teaching it is appalling. This problem is more acute with mathematics instruction than with any other area of academic instruction.

The matter of lacking adequate training in mathematics is aggravated by the attitudes of teachers working in disadvantaged schools. There are many competent, hardworking and dedicated teachers who take on the difficult task of working with slum children. Many of these teachers feel that subject matter holds a low priority for these children. For some, the prime goal is to contribute to the slum child's social adjustment; to 'getting along'. The emphasis is on the development of social skills, most other skills being less important. There are other teachers who feel these children are not going to college and the best education for them is vocational. As a consequence, less intellectual effort has to be expended on them and less effort has to be expended on their intellectual development. Others are so disillusioned by the need to spend a major portion of their time on classroom management problems that they feel the task of teaching is hopeless.

There are, of course, objectively good reasons for many of these feelings. The schools many of these teachers work in were physically qualified for demolition a generation ago. The overcrowding is notorious: 40 children in a classroom is not unusual. State laws controlling the size of classes are avoided through various subterfuges. Classroom management is a constant source of concern with physical violence not infrequent. Taken together with the fact the teacher is bound to have only a rudimentary knowledge of her subject matter the difficulties are obvious. It should be remembered too that the elementary school teacher has to be equally knowledgeable about Science, Social Studies (Geography, History, etc.), the so-called language arts (reading, writing, etc.) as well as be proficient to some degree in art and music instruction.

No matter how beautiful and elegant the mathematics curricula are that are developed they will be paper programs only if no concern exists for the people who will have to use them and the conditions that prevail in the classrooms in which they will be used. SMSG has of course provided special services to teachers, however the training problems with teachers who will work with disadvantaged children will be even greater.

Suggestion 7: Don't be put off by those who hold that the school middle-class culture should not be imposed upon lower class children.

It is very true that the school is a middle-class institution and its teachers reflect it. For good or for bad many of the school's practices and ideals reflect a middle-class orientation. The slum child and his parents face a choice. Either the child acquires the skills and knowledge to make his way in a world requiring middle-class skills or he chooses not to (assuming the community even gives him the opportunity for a choice). The ways of making a decent living without acquisition of these basic skills are rapidly being reduced. Abolishing intelligence testing, or talking 'hip' talk with the kids will not do it. Providing them with adequate educational opportunity, and with the kind of education that will enable them to acquire skills are the things which may keep them from poverty and toward a life with some prospects for self fulfillment. It need not do it through alienating him from his family or his culture. A two-fold opportunity exists - one for him - and one for us.

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DISCUSSION OF BEILIN AND GOTKIN'S PAPER

It was suggested that low achievers may be considered as falling into three groups: those with intellectual deficiencies, those with cultural deficiencies, and those with both kinds of deficiencies. Irrespective of how one divides the population, one cannot ignore the total emotional and social climate of the school in which the curriculum functions.

A question was raised with regard to the transition between stages in Piaget's theory: are there critical ages at which these transitions must take place if they are to take place at all? Beilin said that this was an open question but that he would agree that there are wide differences in the ages at which the transitions occur. More important than grade level for these transitions are the previous experiences of the child. Since it is almost impossible to know the extent and causes of deprivation, one must be optimistic and assume that changes caused by deprivation are reversible.

What are the strategies that should be employed at the various stages, e.g. should one verbalize at the level of concrete operations? Beilin said that this depends upon the type of children being taught. He cited a study by Krutetski in which students with little capacity for mathematics demonstrated uneven development of verbal-logical and visual-image systems. Gotkin observed that some students have difficulty in dealing with line drawings of objects and may more profitably learn if exposed successively to experiences with objects, exact pictorial representations, and then silhouettes.

Since many of the problems outlined in Beilin's paper arise before the child reaches first grade, should we consider the implications of the schools dealing more directly with families? The question of mass instruction at pre-school levels is a serious social issue and would involve a huge investment for society. The alternative is to improve existing programs. Suggestions given for more fully utilizing existing resources included: extending the school day, using school facilities during the summer for nursery and pre-kindergarten instruction, establishing a State

Peace Corps to staff such schools and offering training in docility to prepare students to learn.

Beilin felt that such procedures were more feasible in terms of manpower than dealing directly with the family. Some reasons advanced were that it is almost impossible to get parents to come to schools and that teachers may be hostile to working with parents.

Others felt that we should consider non-traditional ways of involving parents, taking advantage of new techniques worked out by other social agencies.

We should recognize that pre-school skills are the most difficult to handle in group settings. They are not in the curriculum today. We need to discover the kinds of habits of thought to be developed in pre-math instruction.

Francis A. Gregory

MATHEMATICS FOR THE UNEMPLOYED

Experience under MDTA. As we deal with the unemployed of our country it is increasingly clear that lack of mathematics skill and understanding is a significant contributor to their unemployability. It goes without saying that in general, the higher the level of the occupation under consideration, the more significant the mathematics component. This is a major characteristic of the work of the world as technology advances and job requirements migrate upward. I know none is more aware of this than those involved in mathematics education.

Direction of remarks. To meet the purposes of this conference I have chosen to give some account of our experiences in training and retraining the unemployed under the Manpower Development and Training Act with particular reference to the field of mathematics.

Passage of MDTA in 1962. This legislation was passed in March, 1962, but it was not until September of that year that money was made available to begin training. Most of you, I suppose, are familiar with it in a general way. It was the first really serious attempt by the American people to face up to the gravest internal social problem which we have next to the question of civil rights--namely, what to do about the millions of people who are out of work.

The roots of the problem. There is, as you know, a chronic unemployment of about 5,000,000 in this country and there are no indications that it will diminish soon. New machines and automatic systems are displacing 25,000 - 40,000 workers a week, and every year hundreds of thousands of young people drop out of school before completing high school. The economy is growing, but new plants and extensions of old plants quite naturally install the most modern equipment and systems, requiring better educated and trained people; and the burden of unemployment among the unequipped remains great.

What MDTA can do. The Manpower Development and Training Act, which trained 65,000 persons during its first fiscal year of operations and is expected to train 275,000 to 350,000 annually at its peak, will not solve this unemployment problem. All we can hope to do under this Act is keep enough turnover among the unemployed to prevent long-term unemployment from becoming chronic and to demonstrate to the American people the benefits of training.

Awakening of people to need for training. Actually, the greatest benefit of the Act probably lies in just this awakening of our people to the fact that in the 1960's and in the years beyond we are going to have to train and retrain our adult population. The impetus MDTA has given vocational education has been great. The Vocational Education Act of 1963 is another manifestation of this growing awareness of a wide gap in our educational service which must be bridged.

Adult vocational education. All the pupils in our vocational schools today are important--but none more so than the hundreds of thousands of adults who have been out of school for years and have gone back to learn a new occupation or learn new skills in their old occupation. These are the students taking advantage of existing opportunities in adult education. There are machinists learning shop mathematics, housewives learning typing, mothers learning to be practical nurses, glaziers learning metalworking so they can fit glass into their new metal settings, carpenters learning the welding demanded on big construction jobs, and so on. These are not bright-eyed youngsters. These are serious and determined grown people, anxious and eager to learn, sitting side by side sometimes with youngsters, often holding down a full-time job by day and coming into an evening class, tired but determined. They know what the 1960's demand.

Who the MDTA trainees are. These are not the unemployed being trained under MDTA. MDTA trainees may be found in the same institution--often in classrooms next door. The MDTA trainees are usually a little shabbier, a little less well-dressed, a little more careworn and sober, but just as purposeful and just as determined. The MDTA trainees do not live and eat and feed and clothe their families on the rewards of work--they do this on a training allowance which ranges from $15.00 a week in Puerto Rico to $48.00 in some states. The allowance is based on the average unemployment insurance payment in each state. A recent amendment to the law will permit up to $10 a week additional hardship pay. But whatever the amount, it is hardly enough to feed and clothe a household. I might say in passing that only those who are heads of household and have been in the work force two years or more are entitled to allowances, except for certain underprivileged youth who may receive a maximum of $20 a week.

Characteristics of MDTA trainees. Who are these people who take MDTA courses? Let us look at some statistics. Fifty-seven percent of them are male, 75% are white, 12% are under 19, 25% are 19 - 21, 55% are 22 - 44, and 10% are 45 and over. Two percent had less than an eighth grade education, 54% were high school graduates, and 11% had education beyond high school, 45% had been unemployed 15 weeks or longer, and 20% more than a year. We hope to do more training as time goes on in three categories:

1. under-privileged youth;
2. older workers, that is, those over 45; and
3. that vast and illdefined category often called the "hard-core unemployed."

The "hard-core" unemployed. The "hard-core" consists of those people who are unemployable, or nearly so, because of lack of education, lack of skills, lack of ability to communicate with people in any medium, a history of delinquency, or job jumping or something else which makes an employer leary of hiring them. There are more hard-core unemployed among minority groups--Negroes, Puerto Ricans and Mexican Americans--than among the dominant ethnic groups. But there is a large group of white, third-to-fifteenth generation Americans, who are also in the "hard-core". These are the coal miners and marginal farmers of Appalachia who find themselves without jobs, without education, and without hope.

Length of MDTA training. What kinds of jobs are MDTA trainees being trained for? We must keep in mind that the law limits the payment of training allowances to 52 weeks, which effectively limits training to 52 weeks. Under a recent amendment, 20 more weeks can be added if the trainee needs basic literacy training. This limitation of time limits the variety of courses. For semi-skilled occupations, with trainees who do not need a great deal of literacy training, the time is adequate. For skilled occupations, only training for the entry skills or upgrading or refresher training is feasible. It is adequate for most of the office and sales occupations provided and--this is important--the trainees have sufficient educational background to begin with.

A look at our last record of courses completed or in operation shows:

Professional and managerial. From August, 1962, through December, 1963, 315 training projects were approved in categories we label, for want of a better term, "professional and managerial." 'Technical" might possibly be a better word. Of these, 46 were for draftsmen of various kinds, 147 were for licensed practical nurse, 20 were for forester aide, and 15 were for computer programmer. Others were in such occupations as technicians in the health field and civil engineering aides.

Clerical and sales. Five hundred forty-three courses were in the "clerical and sales" category. Two hundred thirty-nine of these were for stenographer or secretary, 138, for clerktypist or typist, 53, for general office clerk and other clerks, and 35, for salesmen and saleswomen of various kinds.

Service. Two hundred twenty-four courses were approved in what we call generally "service" occupations. Fifty-five of these were for chef and cook, 30 for waiters and waitresses, and 66, for nurse's aide. We include chambermaid and janitor and other jobs of this kind under this heading.

Agriculture. Seventy-six courses were in agriculture, including 21 for farm hand, 10, for tree and vine pruner.

Skilled. Now we come to the big category--the skilled occupations--1,056 projects under 175 different job titles. Let me start reading the list to show you the variety--baker, butcher, knitting-machine operator, garment mender, weaver, loom fixer, textile mender, seamstress, tailor, garment alterer, garment cutter, garment inspector, lumber grader, head sawyer, etc.

The skilled occupations which have the most training projects are welders, with 138 projects--automobile mechanic, with 205 projects--automobile body repairman, with 105 projects, and electronics mechanic, with 50 projects.

Semi-skilled. In the semi-skilled category there were 561 occupations, the majority of which were for machine operators of various kinds in production situations. These are the factory jobs.

Pre-apprentice. Fifty-one projects were for pre-apprentices--training courses which would lead to formal apprenticeships--for carpenter, machinist, lithographic artist, and the like.

Unskilled. Only 26 projects were for the unskilled occupations--crab picker, slaughter house worker, woodenware assembler, etc.

Reasons for more advanced courses. You may be asking--why the preponderance of courses that demand an educational background of the kind that most of the "hard-core unemployed" will not have? The answer is--the courses are given where the job opportunities are. There are simply not enough jobs for janitors and porters and chambermaids and elevator operators to go around, and they are becoming fewer all the time. When we train the "hard-core" our employment offices often have to do "job development," that is, go out and sell employers on the trainees. The law requires, by the way, that there be "reasonable expectation of employment" before we start training.

As mathematics educators, you can see at once that mathematics skills are necessary in most of these occupations. We will look more closely at a few of them in just a moment.

Variations in courses. Course length and content vary from state to state and from locality to locality. The Department of Health, Education and Welfare which is responsible for institutional training under the Act, encourages these differences, so that the particular needs of a locality can be met. Besides, we have found that longer courses are more feasible in some places than others, because of differences in allowance payments and economic conditions. Thus trainees in a locality where the allowance is low and odd jobs are plentiful are likely to drop out and take temporary jobs if the course is very long.

Industry committees. The Vocational Education officials who prepare the training plan, or course of study, often work with an Industry Committee which helps them determine the local requirements.

Mathematics. Let us take a look at some MDTA curricula for their mathematics inclusion. A general office clerk course was made up for Nevada which lasted for 400 hours and included 20 hours of basic mathematics--addition, subtraction, multiplication tables, decimals, simple fractions. Twenty hours are not very much, and you can readily surmise that this was intended only as a refresher for people who already had a background in the subject. When Tennessee planned a General Office Clerk course, they included more office machines to make a 1020 hour course, including 35 hours of the same kind of mathematics. They also included 35 hours on the machine calculator, doing the same thing by machine. Do office clerks need more math than simple arithmetic? It is difficult to generalize but I daresay that a clerk who can do the simple computational jobs rapidly and accurately will be found to be a useful addition to any office.

Mathematics for janitors. There perhaps is little real difference between the mathematical subject matter of this course and that of the Wisconsin course for janitor, designed for persons with low literacy levels. Here the text is a standard one, *Growing Up With Arithmetic*, 4, by Weber and Weber. A janitor nowadays has to make simple arithmetical computations, add columns of figures, calculate fractional pieces, multiply, and divide. Granted, he does not have to do it nearly so fast as the office clerk, and the columns of figures are not so long, but essentially the content is the same.

Arithmetic texts. We will be doing more and more of this kind of arithmetical training. You know better than I what basic arithmetic texts are on the market written especially for adults. Our impression is that there is almost a vacuum in instructional materials in basic education to match the interest and achievement levels of the under-educated adult. It is encouraging to learn of new interest on the part of textbook publishers in producing such materials.

Need for mathematics in all courses. When you stop to think about it, in just about every kind of job for which we train under MDTA, basic arithmetic is needed. The waiter has to multiply and add. The saleswoman has to do the same. The nurse's aide and the practical nurse must know how to figure. Even the fry cook, if he is to progress to anything beyond frying hamburgers and eggs, must know how to read and interpret recipes and project them to the size of the group he's cooking for.

Mathematics in skilled occupations. When we get out of the simpler semi-skilled occupations, we almost invariably encounter heavy doses of mathematics. When we look at New Jersey's chemical assistant course of 1300 hours, we would certainly expect to find mathematics perhaps even more than the 70 hours the course crowds in--70 hours of arithmetic, algebra, and chemical math, which includes measurements, compounding, gas laws, percentages, formulas, and proportion.

Mathematics for draftsmen. You would expect considerable mathematics in a draftsman course, even after remembering that a draftsman trainee of necessity would have to have the equivalent of a high school education. Thus Minnesota has an 1800-hour course for draftsmen under MDTA which includes 180 hours in technical mathematics. This is divided into two parts:
The first part is described in the course outline as:

> "Review of the basic mathematics necessary to compute normal patterns in the drafting field. Included are fractions, decimals, percentages, ratio and proportion, elements of algebra, slide rule and the use of tables and handbooks to find area, volume, weights, etc."

The second part is described as

> "Algebra, slide rule and logarithmics in the study of trigonometry."

Computer programmers. In the computer programmer courses of various states, as you might expect there is a great deal of mathematics of a rather advanced type.

Skilled blue collar occupations. We have talked some about the courses with minimal mathematics demands and some about those with major demands. But there is a large group of occupations we haven't discussed--those rugged, skilled occupations usually known as "blue collar" but which require a great deal of skill and training. I am talking about carpenters, plumbers, machinists, and the like. As I said earlier, our training here is either for entry skills or for upgrading.

Upgrading of carpenters. When would we upgrade the skills of say, a carpenter? As an example, in Salt Lake City, outside work on construction projects usually shuts down during the winter months. Most of the carpenters become unemployed. Who are the first ones to be laid off? Those who have only the basic carpentry skills, who haven't the training or the ability to be inside finish carpenters or who don't know how to weld or do elementary metal work. It is interesting to note in our rapidly changing technology of today, a carpenter on heavy construction must know how to weld.

Mathematics for carpenters. So--MDTA in Salt Lake City pays for a 180-hour upgrading course for unemployed carpenters in the winter months, teaching them welding, metal work, principles of surveying and site preparation, stress and strain, how to estimate and prepare bids, *and mathematics.* Ten hours are devoted to review of the fundamentals of math and ten hours to formulas and problem solving. Then there is mathematics in the training they get in surveying, preparing bids, and the use of the steel square. Square root is important in teaching the steel square.

Math common to several skilled jobs. In many skilled occupations men have to know how to figure circles, squares, rectangles, angles, triangles, and the like--in addition to carpentry, there are tilesetting, ship-fitting, bricklaying, ornamental iron working, sheet metal working, landscape gardening, dozens of others.

Need for practicality in teaching. I could go on. But I think you get the idea. And I suppose I hardly need to say that the mathematics taught these people, many of whom have not been inside a school for years, must be of the most practical kind.

"Many have not been inside a school for years"--10, 20, even 30 years. How can they be taught mathematics?

Ability of older trainees to learn math. The answer may be a surprising one, but our evaluators from the Office of Manpower, Automation, and Training, who travel all over the country and talk to instructors and trainees, tell me it is true. Older people can be taught the mathematics they need as easily or more easily than the younger ones.

Reasons for ability of older trainees. This is not to say that a middle-aged carpenter, twenty years out of school, can learn a new mathematical concept quicker than a recent high school graduate. But I am saying that the 40-year-old carpenter, with the intense motivation he has and the ability acquired over the years to stick to a task can and probably will learn that mathematics more thoroughly and remember it longer than the twenty-two-year-old beside him. The instructor may have to use special methods--in one school the supervisor of the courses took a group of men who were having trouble learning mathematics in the classroom and put them in a work situation with real rafters and boards to figure on and they learned in a hurry, faster than those who remained in the classroom--but the course doesn't have to be made easier for the older ones.

Motivation. Don't forget too that these people have had years of experiencing the importance of mathematics in their daily living. They can see its importance in a way that the youngster cannot. They have probably regretted hundreds of times that they weren't better at figuring things. The learning is like a revelation to them, and they devour it.

The learning experience and the unemployed. Not at first--they dread it. They go to their first classes with fear and trembling. And then they find often that school is fun--something they never realized when they were younger. The rich fulfillment that only a learning experience can give comes with a peculiar poignancy to the unemployed, who probably would not have been unemployed had they experienced the same delight in learning when they were youngsters.

These readings on learning capacity for the under-educated adult suggest the fallacy of generalizations on the hopelessness of working with the "hard-core" of the unemployed.

Conversation with trainee. Our evaluators report some interesting conversations with some of these trainees. Here is a sample of part of a typical one:

Evaluator: Mr. Jones, how many years has it been since you went to school?

Jones: Nigh onto twenty-five years, I reckon.

Evaluator: How do you like going to school again?

Jones: Well at first I was kind'a scared of the whole idea. I was afraid I wouldn't do so well along with them younger fellows. But it didn't take me long--and now I'm keeping right up with the best of them.

Evaluator: Have you had difficulty with any particular part of training?

Jones: Well yes, all that math kind'a floored me. But I kept pluggin', and at night me and my wife and my oldest boy--he's in high school--we'd go over everything, and pretty soon I found myself enjoying' it, and now I can truthfully say the math is the most interestin' part of the whole business. I didn't like it in school, but I never knowed what I was missin'.

Math for the recent dropout. Teaching mathematics to the recent dropouts is a different story. They haven't been seasoned by experience, and their memories of school are unpleasant. They are convinced they cannot learn or that learning is drudgery. They particularly dread and fear mathematics. They have no realization of the importance of math. This makes it all the more important that their curriculum, their texts, be written for the most practical applications.

Home environment and math. In developing mathematics courses for the unemployed, the developers must keep in mind that the home environment of these people is often not conducive to homework. Most of them do it through the noise of television and squalling babies. Trainees for licensed practical nurse, often widows, have a substantial amount of homework, including math, which they always seem to do. The weatherbeaten carpenters of Salt Lake took a great deal of pride in doing their book-work beside their children. But it isn't practical to expect them to do very much.

Opportunities for development work in curricula. I would say this training of the unemployed in mathematics offers a wonderful field for research and experiment. The vocational educators who develop our courses know their business, of course, and do a fine job of adapting. But I am sure they would welcome developmental work in mathematics curricula for adults with little skill or little recency of skills.

That there is room for development of new methods and materials for teaching adults no one can seriously question. This is part of the unfinished business of public education in the United States.

DISCUSSION OF GREGORY'S PAPER

The comment was made that the papers of Gregory and of Beilin led to opposite conclusions as to whether or not the family is unreachable by the school.

In reply to a question as to whether the government had given thought to combining the problem of retraining adults and children, Gregory observed that omnibus education bills have had an unfortunate legislative history. Thought has been given to such problems by the Office of Education, and the experience of the Ford Foundation and the Greater City Schools Improvement Project of Washington, D. C., has demonstrated the value of a total approach.

Beilin felt that his position was not incompatible with that of Gregory; however, Beilin questioned the extent to which parents could be trained to act as teachers of mathematics.

It was pointed out that the white collar workers discussed by Gregory and the lower class minority groups discussed by Beilin presented different problems. Although the characterization of urban families as not intact and disinterested in school was challenged, it was agreed that family background is a crucial consideration in curriculum planning.

Although some felt that considerable mathematics and reading instruction is done at home, others suggested that it is not typical for parents to sit down and help their children with school work and that materials should be devised so that the learning could take place almost entirely in school.

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Gloria F. Leiderman

MENTAL DEVELOPMENT AND LEARNING OF MATHEMATICS IN SLOW-LEARNING CHILDREN

While problems of teaching slow-learning children have concerned us for many years, they have become more pressing with the development of new curricula in mathematics and the physical and biological sciences. If these new curricula are not to be restricted to a limited segment of the school population, we must examine the relation between the ability to learn concepts in mathematics and the sciences and intelligence, age, and stages of mental development of the child.

In this paper I shall deal with one aspect of this problem: the relationship of a child's mental development to the learning of mathematical concepts.

If we consider the slow-learning child as one with limited abilities and potential for learning, then our emphasis will be on creating learning experiences suitable for his circumscribed capacity. However, if our approach assumes that the child is a changing organism whose development is affected by both external and internal factors which may expedite or impede his development, our thinking and planning for his education will consider his motivation for learning as well as his cultural environment. I assume that eventual understanding of the slow-learning group demands consideration of the development of intelligence and the elements contributing to its effective utilization. Toward this end, literature in many fields has been searched, ranging from studies of mental development and motivation in children's learning to the problems of children from special groups in the population and studies of specific educational programs for slow-learning children.[1]

A typical definition of the slow-learning group is as follows: "The slow learners are those children who score between approximately 75 and 90 I.Q. on a verbal intelligence test.

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[1] The problems of those children retarded as a result of organic damage or trauma to the central nervous system are considered to be qualitatively different and will not be dealt with in the present paper.

Translated into comparative intellectual growth terms, this means that they are developing at a rate between three-fourths and nine-tenths that of the average rate" (Johnson, 1963, p. 42). This definition separates the slow-learning group from the more severely retarded as well as from the normal range of learning ability. It further sets limitations on both rate of development and maximum performance. Another criterion frequently used is retardation (approximately two years) in school performance as measured by achievement test scores and grade placement for a given chronological age. These are useful indices, giving a common frame of reference for considering this special group.

Many arguments can be summoned against using the I.Q. concept as a major criterion for evaluating ability, and these will be discussed in a later section of this paper. Work in the area of mental developmental processes is most relevant to understanding how some children become "slow learners" and whether their learning differs qualitatively from children who learn more rapidly.

The prolific work of Piaget and his collaborators over the past thirty years has been ably reviewed and synthesized with other theories of mental function by Hunt (1961). He points out that one of the main themes of Piaget's formulations is the operation of adaptation to outer circumstances and the progress of inner organization as a result of the child's interaction with his environment; "experience, defined as the organism's encounters with his environment, is continually building into the developing organism a hierarchy of operations for processing information and for coping with circumstances" (Hunt, 1961, p. 246-247). The child's experience with a wider range of experiences as well as more varied situations forces him to modify his behavioral structures, which, in turn, increase his interest and his ability to respond in more flexible and varied ways.

Luria and Yudovitch (1959) emphasize the role of speech as a central factor in mental development. "Inter-communication with adults is of decisive significance because the acquisition of a language system involves a reorganization of all the child's basic mental processes; the word thus becomes a tremendous factor which forms mental activity, perfecting the reflection of reality and creating new forms of attention, of memory and imagination, of thought and action" (p. 12).

Several significant elements relevant to the slow-learning group may be abstracted from the brief descriptions of the work of Luria and of Piaget. One of these concerns the importance of the early experiences of the child with both the physical and social qualities of his environment. Another is the significance of the development of language ability. A third and fourth which cannot easily be partialled out from the first two are the quality of interaction with adults as a factor in the young child's mental development and the importance of variety of stimulation and experience.

One additional source for formulating the impact of early experiences on cognitive development will suffice to set the stage for the major thesis of this paper. Bruner (1961) states that, "Not only does early deprivation rob the organism of the opportunity of constructing models of the environment, it also prevents the development of efficient strategies for evaluating information--for finding out what leads to what and with what likelihood. Robbed of development in this sphere, it becomes the more difficult to utilize probable rather than certain cues, the former requiring a more efficient strategy than the latter. Then there is the matter of the unsuccessful attempt to order the unorderable environment perceptually" (p. 202-203). The development of a model of the environment is of primary importance. It conserves information in the form of concepts; it distinguishes predictable from chance events; and it makes possible the ability to use inferences which demand 'going beyond the information given' (p. 200).

The major thesis of this paper, derived from developmental approaches, is that the slow-learning group consists of some children, perhaps a majority, who are retarded in their mental development because contact with their physical and social environment was deficient in their early years. This thesis makes research data on the culturally deprived groups relevant. It also brings into a new focus the many studies of the characteristics and achievements of the "retarded" or slow-learning pupils in the school population.

Several hypotheses may be deduced from this theoretical position. One is that a deficiency or deviation in verbal development during the early years will have adverse effects on the child's later ability to solve problems. Specifically, those children whose verbal ability is limited through these deprivations can be predicted to be less capable in mathematics. A second hypothesis concerns the ability to abstract, to use symbolic cues for prediction purposes. If a child has been deprived of contact with the kind of environment necessary to develop these inner cues, hypotheses or signals, then his ability to handle problem-solving tasks will be limited.[2] He will need to depend more upon concrete events and properties of a situation rather than being readily able to abstract common elements and make predictions.

A third hypothesis is that children coming from more culturally disadvantaged groups in the population will contribute in larger proportion to the slow-learning groups than will children from more advantaged groups.

Studies of slow-learning or retarded children can be used as indirect evidence for testing the above hypotheses. The limitations of such an approach are clear, but, as a preliminary to direct studies of such hypotheses, these studies of retarded children may provide suggestive evidence and guides for research.

The first hypothesis stated that those children whose verbal ability is limited will be less capable in mathematics. Thompson and Finley (1962) in a recent study of intellectual patterns of gifted and mentally retarded children used the sub-tests of the Wechsler Intelligence Test for Children to show the areas of intellectual strengths and weaknesses of these two groups. The mean Full Scale I. Q. for the gifted group was 132.51 (N = 400), for the retarded group 67.74 (N = 309), chronological age 10.23 and 10.91 respectively. Weighted sub-test scores were rank ordered for each group. The retarded group ranked highest in the picture completion, block design, and object assembly tests and were lowest on the information, vocabulary, and arithmetic sub-tests.

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[2] It might be added that the kind of contact with the environment is a significant influence. This variable would lead into dimensions of the child's organic structure enabling or limiting his utilization of environmental interactions. Another direction it might take would be the quality of infant-mother relationships in the early years which is beyond the scope of this paper.

On the other hand, the gifted children were highest in similarities, information and comprehension and lowest on the object assembly, picture arrangement, and coding sub-tests.[3]

If reading comprehension and vocabulary are used as indices of verbal ability, the findings of studies on the relationship between reading and mathematics achievement are pertinent. Dunn (1954) compared a group of twenty mentally retarded boys, selected from special classes, with thirty normal elementary school boys on certain aspects of the reading process and factors related to it. The two groups were not different in mental age (mean MA for the retarded 9-2, normals 8-11) but differed significantly on chronological age (13-3 retarded, 8-11 normal) and I. Q. (mean I. Q. retarded 70.55, for the normal group 101.77). Although there were no significant differences between the two groups on reading capacity or on arithmetic fundamentals, the normal group was significantly superior to the retarded on both silent and oral reading and arithmetic reasoning.

Eagle (1948) and Pitts (1952) both studied the relationship of reading abilities to competence in mathematics, the former using ninth grade students and the latter 11th grade girls. Even though they used different tests to measure reading levels and mathematics success, their results are corroborative in finding reading comprehension positively related to success in mathematics. However, it is the association of reading level and mental age which most influences the correlation between reading and mathematics competence.

Cruikshank's (1948) study of the ability of children to separate extraneous material from the arithmetic facts needed to solve a problem is also relevant to this hypothesis. His subjects were two groups of fifteen boys each, matched on mental age and arithmetic age. The I. Q. of the experimental group was 73.33 (C.A. 14.29) and of the control group 110.4 (C.A. 9.09). Eight sets of arithmetic problems, each set consisting of three problems were presented individually. Problem "a" in each set contained a great amount of superfluous, extraneous material in addition to the material needed for problem solution. Problem "b" contained only the verbal material specific to solving the problem. Problem "c" consisted only of the problem set up in computation form. The control group was significantly superior to the experimental group in the number of problems solved correctly on all three series (a,b, and c). The greatest difference between groups, however, was on series "a" in which it was necessary to select out the pertinent facts from considerable extraneous material. Cruikshank attributes this difference to the inferior language development of the retarded group and to their more limited understanding of arithmetic vocabulary.

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[3] It should be pointed out that there was no overlap in the distribution of mean sub-test scores for the two groups.

The second hypothesis stated that if a child has been deprived of contact with the kind of environment necessary to develop inner cues, then his ability to handle problem-solving tasks will be limited. As evidence for this hypothesis, two sources of data can be cited. The first source is empirical studies using animals, and the second, reports of development of children in institutional settings.

Thompson and Heron (1954) used dogs to test the effects of early experience on later problem-solving. Thirteen pups were submitted to three degrees of restriction of early experience: two of the pups were reared in cages in complete isolation from other dogs or human beings from the time they were weaned until they were eight months old. Eight were reared for the same time period in laboratory cages which had wire-mesh sides and fronts covered with cardboard. The remaining three were reared in the laboratory in open wire-mesh cages. Each of the cage-reared dogs had a litter mate in the group which received "enriched" experience by being reared as pets in homes from their weaning till eight months old. The problem-solving tasks reported were given at approximately eighteen months of age when the two groups of animals had had ten months of similar experience living in the laboratory. Both in orientation tests (finding food behind a screen in a different corner of the room from the one in which they had several experiences of finding it) and barrier tests, the pet-reared dogs were significantly superior to the cage-reared in modifying their habit of running directly to reach food. In a delayed-reaction test in which the dogs were forced to wait various periods of time before being released to obtain the food they had watched being placed, the pet-reared dogs were able to tolerate long delays and find the food in many fewer trials than were the cage-reared.

These findings have interesting implications for the theory that greater stimulation and variety of experience in the early years permit more flexibility of response through the development of central mechanisms, previously referred to as sets or cues.

Studies of infants and young children growing up in institutional settings (Dennis, 1960, Spitz, 1945, Spitz, 1946) are in agreement about the retardation evidenced in behavioral development where the environment offers a minimum of stimulation. The lack of handling, and of opportunities for motor experiences, the paucity of visual stimulation through toys to watch and later manipulate, the infrequency of adults talking to them, are all considered contributing elements to the retardation. However, there are few follow-up studies to offer direct evidence for our hypothesis of the effects of these kinds of early deprivations on problem-solving behavior in children.

The one directly relevant study on children is that of Goldfarb (1943) who followed into adolescence matched groups of institutionally-reared and foster-home placed children. The institutional group consisted of fifteen children ranging in age from ten to fourteen years (mean age 12-4), who had entered the institution in early infancy (mean age 4.5 months), had remained in the institution for about three years, and had then been transferred to foster homes where they had been reared up to the time of the study. The fifteen foster-home children had always lived with a family: with their own families until a mean age of fourteen months, and then in foster homes. The age of the foster-home children at the time of the study was ten to fourteen years (mean age 12-3). The investigator was interested in personality and intellectual differences between the groups. On the Wechsler-Bellevue Intelligence Test, the performance of the two groups differed significantly with the mean I. Q. of the institution group 72.39 and of the foster-home group 95.37. In order to evaluate the concept formation ability of these groups, several measures were employed. On the Weigl Color Form Test, the Vigotsky Test requiring the subject to sort blocks into groups on the basis of a principle which the subject has to discern, and two sub-tests of the Wechsler-Bellevue, Block Design and Similarities, the foster-home group was consistently and significantly superior. From his findings, Goldfarb states, "We have striking confirmation of the hypothesis that the abnormal concretivity of the adjustment of the institutional children is more than a reflection of low intelligence. There appears to be an unusual primitivization of the total personality, which is also manifested in the perceptual and conceptual development of the institution children" (p. 116).

One can question the findings of this study on methodological grounds since the initial matching of the children in the two groups may not have been sufficient to insure comparability. The work of other investigators, previously cited (Dennis, 1960, Spitz, 1945, Spitz, 1946), demonstrates clearly the profound differences which do exist from one institution to another, and there is no attempt in this article to describe the institutional setting. In addition, the quality of the foster homes, or their provision for experiences which would stimulate or compensate for early deprivation, may be considered insufficient. However, these findings are clear in supporting the hypothesis relating early deprivation to limitations of later problem-solving and abstraction abilities.

The third hypothesis stated that children coming from the more culturally disadvantaged groups in the population contribute in larger proportion to the slow-learning group than do children from more advantaged groups. Literature on the Negro group indicating that Negroes are retarded in arithmetic, reading, language usage, and ability to handle abstract concepts is abundant (Bullock, 1950, Osborne, 1960). "The extreme intellectual impoverishment of the negro home *over and above* its lower social-class status reflects the poor standard of English spoken in the home and the general lack of books, magazines, and stimulating conversation. In view of the educational and psychological inequality of segregated schools, the inferior intellectual status of negro homes, and the negative motivational aspects of membership in a socially stigmatized group, any inferences from the lower I. Q.'s and educational retardation of negro pupils regarding *innate* differences in intelligence are obviously unwarranted" (Ausubel and Ausubel, 1963, p. 125).

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In his study of the effects of social stratification upon academic achievement of elementary school children in the Berkeley, California, school district, Wilson (1963) found that children in the upper strata schools were far superior to those in the middle strata, and those in the middle strata superior in achievement to those in the lowest strata schools. His division of the school district into the three social strata followed the lines of residential segregation in the community, with the highest social strata schools located in areas in which predominantly white, college-educated, professionals and executives reside, and the lowest strata schools located in sections heavily populated by Negro manual workers. The indices of achievement were performance on reading and arithmetic tests, I. Q., per cent of children reading at grade level, and per cent receiving A or B in reading and arithmetic.

Obviously, the Negro group is not the only one in our population representing the kinds of cultural disadvantages with which we are concerned. Literature on this group has been referred to specifically for two reasons. First, Negroes comprise a sizeable minority within the U. S. population whose status demands immediate attention, and, second, they represent a kind of prototype for other deprived groups.

To summarize the arguments thus far, we have brought together data from many sources to test the meaningfulness of the theoretical approach presented. It seems reasonably clear that early deprivation in the child's environment will result in less ability to abstract; that his lessened verbal development is related to difficulty in learning mathematics; and that a larger proportion of children from culturally disadvantaged groups in the population contribute to the slow-learning and low-achieving groups in our schools.

I would further suggest that this theoretical view helps in understanding the problems of cumulative deficit and cumulative failure which are of ever-present concern to educators dealing with slow-learning children. If the child has not learned the necessary operations and symbolic cues during the course of his pre-school development, through variety of stimulation leading to both finer discriminations of relationships between things and broadening of categorizations, then a school program predicated on this basic development will not be successful. If the early rudiments for concept formation or abstract reasoning are lacking, then these tasks, as the child progresses, or, more accurately, is passed on from one grade to the next, are not only inherently more difficult, but are more difficult because the discrepancy between the child's ability and the environmental expectancies is continually increasing. It appears that the possibility for learning depends on a "match" between the child's abilities, which are dependent on readiness or developmental stage, and the tasks expected of him. If this position is accurate, then trying to teach the child by drill, presenting the material at a slower rate, or in simplified language, all of which have been recommended, are not sufficient for the planned learning to occur. It might be noted in this context that Pitts (1952) had predicted in his study that older subjects would perform better than younger on his test of functional competence in mathematics. This prediction was based on the assumption that the older girls, having had more experience with the kinds of tasks measured, would be superior in functional competence through using mathematics in their everyday experiences. This prediction was not supported; a negative correlation between chronological age and test performance was obtained.

John's study (1963) on the verbal development of children from an urban Negro slum area illustrates this point well. She used first and fifth graders in three socio-economic levels to test several dimensions of verbal and classificatory behavior expected to be most affected by the social experiences of the child:

1. Verbal behavior of both an enumerative and of an integrative type (intergrating parts of a picture by giving the most appropriate title to that picture), in which little difference was expected on enumeration, but greater difference in favor of middle class children on integrative behavior;
2. Word associations in which a higher percentage of form-class responses (noun associated to noun rather than phrase completion responses) and shorter latencies were expected to be given by children from the more

advantaged social groups; and

3. Sorting behavior in which middle class children were predicted to give concept-sorts accompanied by explicit verbalizations of the rationale for classifying.

Significant differences in the predicted direction were found for fifth grade children on integrative tasks as well as on conceptual sorting and explicit verbalizations of their sorting behavior. Differences at the first grade level were not significant although in the predicted direction. The finding of most direct interest for the present discussion, however, is the large intra-group variability on integrative behavior at the first grade level, with much greater homogeneity within social classes evidenced by the fifth grade level. John interprets this stabilization of responses within a group from first to fifth grade as reflecting the action of cumulative, common social influences on these children. I would broaden this interpretation to suggest that the significant differences found at the fifth grade between the children of lower and middle social classes represents also the cumulative effects of progression through school for the lower class children without the necessary early learning upon which the continued development of this integrative ability is based. Thus, the gap, for the larger number of lower class children, between ability and expected performance becomes ever larger.

The concept of cumulative failure is difficult to separate from that of cumulative deficit. If, for example, the child has not learned the fundamental operations of addition, subtraction, and multiplication in the primary grades, then he is not only likely to get "farther behind" when the concepts of fractions or decimals are introduced, but also, "Learning becomes less and less probable as continual failure dulls any remaining interest on the part of the child" (Krulik, 1961, p. 8). Dunn (1954) noted in both his sample of retarded boys and in his review of relevant literature that pupils in special classes are not only retarded in reading achievement but do not read up to their mental age expectancy level.

Clearly, then, the ideas of cumulative deficit and cumulative failure, albeit not independent, are separate phenomena, cumulative deficit referring to the increasing discrepancy between achievement and expected learning, and cumulative failure referring to decreasing success and, probably, decreasing motivation.

If we are to assume that some therapeutic work is possible with the slow-learning group, then knowledge of what can be used to motivate these children is significant. An inference to motivation can be made from the theory of limited development of abstraction ability here suggested. If these children have not developed to the level of their more advantaged peers in handling abstract concepts, can we expect them to respond to abstract rewards for learning? The values of "being right" and of acquiring knowledge for its own sake are more typically taught by the middle class group to their children, and these highly abstracted values may not serve as sufficient reinforcers for learning among children of other socio-economic classes. The work of Zigler and de Labry (1962) and Zigler and Unell (1962) has shown that significant differences in ability to switch concepts can be achieved when tangible reinforcers (in these instances, toys) are used for retarded and lower class children; whereas the performance of middle class children is superior to that of the other two groups with an intangible reinforcement condition, i.e. being told they are right.

Not only is the abstract or intangible quality of the reward an important one in school learning, but certain delays in reward are also demanded. Not only does a child have to value receiving a "superior" or an "A" for his efforts and performance, but he must also wait for that gratification until the end of the marking period. Mischel and Metzner (1962) have shown that preference for a larger delayed reward over a smaller immediate reward in elementary school children is positively related to age and intelligence, and negatively related to the length of the delay interval. These findings are useful in pointing to significant variables which may be relevant to the school learning situation, although the specific findings raise more questions in the context of the present discussion than they clarify. The subjects used in this study are described as being from predominantly lower-middle and lower class urban backgrounds, and attention is not turned to class differences on ability to delay reward. Moreover, the reward in both the immediate and delayed conditions was a tangible one, e.g. a larger or smaller candy bar.

Such studies are valuable for our present concerns in suggesting possible motivational variables to consider in a developmental approach to school learning. Certainly dependency, as it is developed in the parent-child relationship and, in turn, carried over to the teacher-pupil relation, and the development of achievement motivation are two such variables among many in addition to those already briefly discussed, but they are beyond the scope of this paper.

Numerous efforts have been made by educators to understand slow-learning children in order to plan more effective teaching programs in mathematics for them. It is important to abstract common elements from these various plans to evaluate what knowledge of children, particular kinds of classroom instruction, and motivational variables have been found useful in actual practice. The work of Mallory (1939), Weiss (1941), a study by the National Education Association (1943), Jewett and Hull (1954), Potter and Mallory (1958), and Krulik (1961), are drawn upon for the present purpose.

The implications of previous failure for motivating children in the slow-learning group have been stressed. There is consensus on the importance of presenting a variety of materials which will interest these children, which will enable them to achieve a measure of success, and which in turn will lead to a more positive attitude toward mathematics and other school subjects. Ability groupings or separate tracks for more and less able students have been utilized in order to better plan material for the achievement level of each group and to minimize feelings of failure by the slower students.

The emphasis on concreteness and on engaging the pupils in activities such as construction of geometric models, participation in discussion, or obtaining information in their community are recurrent themes. The importance of relating mathematics to the present and future lives of pupils is expressed through teaching the social uses of mathematics, such as understanding of statistical concepts and coordinate geometry (reading news articles, using road maps).

The role of the teacher in providing individual guidance, in planning and supervising small units of work with clear explanations and examples, and omitting long lectures, are considered essential since these children are less able to plan and evaluate their own work. The importance of using drill and working without the pressure of time limits are somewhat less emphasized but do receive attention in a number of these studies. It is also relevant to note that, in many of these studies, the variability within the slow-learning group was mentioned as was its over-representation by lower socio-economic groups.

The emphases made in these special programs do not seem very different from what could be expected of planning for any group of children, with but two exceptions:

1. The need to motivate these children for school learning, and
2. The stress on concreteness.

In addition, the variability within the slow-learning group and the number of children from culturally disadvantaged groups, both of which are recurrent themes, are relevant to the theoretical position of this paper.

The statement was made earlier in this paper that arguments against using the I. Q. as a major criterion for evaluating ability could be summoned. To use the I. Q. as a valid predictor of future performance, one of two assumptions must be made. The first assumption is that the I. Q. is genetically based and, therefore, quite inflexible to the impact of varying environmental experiences. The other possible assumption in using the I. Q. at one point in an individual's development to predict his future performance is that the kind of experiences of this particular organism with his environment which have produced the present measured I. Q. will not change radically. Both of these assumptions have a static quality and provide a "set" to the persons using these scores, in this instance, teachers; and this set may minimize possibilities for improved performance. Adler (1963) presents this point of view in arguing for extending new mathematics curricula to all groups rather than limiting them to the more able students. He states that those who rely on the I. Q. theory assume that most children are not capable of learning much, that, in consequence, not much is actually taught these children, which, in turn, leads to cumulative retardation and less capability of learning. This argument, then, is not in opposition to the I. Q. as a measure of the child's current mental functioning as much as in opposition to its use as a predictor of future performance.

Another argument against using the I. Q. is that although it provides a measure of mental functioning, it neither establishes the nature of retardation when such exists, nor does it assist in understanding how this retardation developed. Although I. Q. tests do not purport to measure determinants of mental functioning or motivational factors affecting it, the too common useage of the I. Q. as a label tends to de-emphasize the need for understanding these other related variables.

One further limitation of the I. Q. is in its useage as a generic term, instead of considering it as a single score, obtained from the administration of a specific test which has certain characteristics, i.e. reliability, particular standardization populations. The knowledge that some tests are highly dependent upon verbal skills, others upon certain cultural experiences make the practice of using an I. Q. score, without reference to or knowledge of the test used to obtain this score, highly questionable.

These arguments made against the use of I. Q., although somewhat belabored, serve the purpose of raising questions about reliance upon a single set of criteria for planning school curricula. It is clear that I. Q. scores derived from certain tests have, in the past, been good predictors of school achievement. These same scores may not be good predictors of achievement with the newer curricula currently being tried, nor with methods such as programed instruction. With the point of view that some children are retarded through limitations in their early experiential development, it is likely that programs can be planned which make meaningless the limitations of performance or rate of development implied in the I.Q. concept.

A relevant and highly suggestive study is that of Jensen (1963) on learning ability in children. The author used a direct test of learning to overcome individual differences in initial ability and to assess, by objective evidence, that the subject understood the requirements of the task. The subjects were junior high school students: a group of thirty-six retarded children in special classes, with a mean Stanford-Binet I. Q. of 66.17 (range 50-75); an average group of twenty-four children with a mean I. Q. on the California Test of Mental Maturity of 103.04 (range 90-110); a gifted group of thirteen subjects with a mean Stanford-Binet I.Q. of 142.54. The task consisted of learning, by trial and error, to associate five or six different stimuli (colored geometric forms) with five or six different responses (pressing a particular button in an array of push buttons). Reinforcement was a green light switching on when the correct button was pushed. The first task instructions were adequate for the average and gifted subjects, but it was not certain that all the subjects in the retarded group understood the instructions. Therefore, a sequence of special procedures was carried out with each of the retarded subjects until the subject evidenced learning. These procedures were, first, verbal reinforcement by the experimenter (in the form of "Good," "That's right,") when the subject pushed the correct button.

For those whose performance did not rise beyond the chance level of correct responding with the verbal reinforcement, a second procedure was used. This was "stimulus naming" in which the subject was required to name each stimulus (e.g. red circle) until he could name each stimulus accurately, and then the learning task was begun. All but three children learned after this procedure. For those three subjects, another equivalent task was given the next day with the third procedure which was the same as "stimulus naming" except that the child was required to continue naming each stimulus before pushing the button throughout the series of trials. A fourth procedure was used for two of the children who still did not appear to learn. This was "delayed response following reinforcement" which had been shown spontaneously by many of the average and gifted subjects. These two retarded subjects who had not yet learned the task showed no delay tendency; therefore, when they pushed the correct button, the stimulus and reinforcement light were held on before changing to the next stimulus. Both subjects began learning with this procedure.

The "last test" consisted of a new set of stimulus-response connections to be learned, and the subjects in all three groups were treated identically. The results show that, on the first test, each of the groups differed significantly from each of the others, with the retarded group far below the average, and the gifted group above the average. The retarded group improved significantly on the second test, in which verbal reinforcement was added. On the last test the groups still differed significantly, but the retarded group showed a marked improvement. Much greater variability in scores was shown within the retarded group than within either of the other two groups. However, the variance in the retarded group decreased significantly from the first to the last test while the other two groups showed no change in variance. Other important findings of this study were the large, consistent individual differences in speed of learning. The four fastest learners in the retarded group were all above the mean of the gifted group on all the tests.

In discussing these findings, Jensen points out that learning ability is probably not a unitary ability, but rather consists of a number of relatively independent dimensions. The fact of retardation is shown by the I. Q. and is usually apparent to the child's teacher; however, the specific nature of the disability must be determined by a variety of learning tasks which tap different dimensions.

An alternative hypothesis which Jensen suggests for interpreting his findings is that the normal and fast learners in the retarded group"-- are children who, at some crucial period in their development, have failed to learn the kinds of behavior which are necessary as a basis for school learning and for the acquisition of the kinds of knowledge and skills tapped by I. Q. tests. School learning probably depends tremendously upon transfer from previously learned habits of verbalization and other symbolically mediated behavior. The habit of making verbal responses, either overtly or covertly, to events in the environment seems to be one of the major ingredients of the kind of intelligence that shows itself in school achievement and in performance on intelligence tests. Without this habit, even a child with a perfectly normal nervous system in terms of fundamental learning ability, will appear to be retarded, and indeed is retarded so long as he does not use verbal mediators in learning." (Jensen, 1963, p. 138)

The position taken in this paper has been that intelligence, defined as the ability to utilize experience through symbols, make inferences, and to solve abstract problems, is dependent upon certain organism-environment interactions during the course of the infant's and young child's development. Studies have been cited which support the position that slow-learning children, equated with the upper range of retarded children, are deficient in verbal and problem-solving ability and are more likely to come from families of culturally deprived groups. A brief summary of the work of educators in dealing with slow-learning groups and a critique of the uses of the I.Q. have been presented.

Some limitations to and criticisms of the position I have taken are obvious. One is that there have been no criteria suggested for delineation of those children retarded through experiential deficiencies from those who, through genetic limitation or other central nervous system metabolic defect, could not be expected to profit from changes in educational planning based on the view presented. Another limitation to the position taken is based on the "critical period" concept. The critical period hypothesis states that there are time periods in the development of a species during which certain kinds of circumstances will have effects on the organism but these same circumstances will not have these effects when they occur at another period. We do not yet know what the critical periods for particular kinds of effects, in terms of learning, are for humans. Both of these limitations suggest possible and fruitful research directions.

A question about the point of view presented in this paper and of the arguments summoned to support it could be raised: Is this approach an argument for the imposition of a middle class way of learning and value system on all groups within the population? My answer would be emphatically in the negative.[4] The need for knowledge, which we can assume can be most successfully learned through the schools, is becoming ever more imperative in our rapidly moving and highly technological society. The important factors are those that will contribute to our understanding of why some children, and more likely lower class children, do not learn well. Riessman (1963) points out that there are a number of positive features of lower class culture which tend to be overlooked. Some of these are slowness of intellectual performance, a physical style of learning, and verbal ability which is not expressed in the school situation. It seems highly reasonable to utilize the strengths of a group, through knowledge of its culture and values, to assist in teaching its children more effectively. Research in the areas of cognitive and perceptual styles is clearly suggested.

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[4] It is pertinent to note, however, that education has provided an important channel for upward social mobility in our society.

There are several directions which research might fruitfully take to answer questions about the learning process and mental functions. It may be that there are systematic differences in reliance on particular sensory modalities for the greatest learning. Riessman (1963) points to a physical style of learning in the lower classes. Perhaps proprioceptive and kinesthetic modalities must be utilized more in teaching some children, while the visual or auditory are more significant modalities for others. The variability of performance within the slow-learning group, which has been emphasized throughout this paper, may be related, in part, to such individual differences in modality dependence.

Another significant research direction would be to isolate and define the necessary and sufficient conditions for the development of symbolic, abstract learning. Reference has been made throughout this paper to conditions in the child's environment which may stimulate or limit his development of inner cues or hypotheses; however, these conditions have not been spelled out in detail. Such conditions as the amount and quality of speech directed to the young child, opportunities for manipulating objects, and for variety of visual experience appear to be a few of many likely leads. The possibilities of programs designed to compensate, during the school years, for early deprivation must be considered as a parallel course while knowledge of the earlier contributing factors is being gained. It is imperative that we learn more about the development of thought processes in children before effective planning of specific curricula for such special groups as slow-learning children can be undertaken.

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Llayron L. Clarkson

ON SOME PSYCHOLOGICAL PROBLEMS, SOCIOLOGICAL PROBLEMS AND OTHER DETERRENTS AFFECTING THE MATHEMATICAL EDUCATION OF MANY NEGROES IN THE SOUTH

Thank you, Dr. Begle, for including me in this conference. We address ourselves here to some of the psychological problems, sociological problems and other deterrents affecting the mathematical education of many Negroes in the South. This meeting is in specific recognition of the fact that the proper preparation in school mathematics is essential for each informed citizen in our age and that at the present time there is a gap between true competency and recommended competency in the mathematical facility of many Americans, and again, especially Negroes in the South.

We start our discussion by defining the educationally deprived as those groups of individuals who have not benefited from aspects of middle class culture such as books, education, formal language, etc. It follows that many members of the lower socio-economic groups are educationally deprived and that large portions of these groups represent the theme of our discussions today. Hence, we arrive at the first apparent fact that was in my mind throughout the preparation of this manuscript; that is, many of the low achievers belong to some lower socio-economic group.

Other pertinent facts are apparent. One is that most Negro children are being taught by Negro teachers and this fact will probably remain constant for many years to come. Also, because of the great effort of many users of mathematics in industry and government toward making certain occupational opportunities known to the Negro college graduate, few of the really talented college graduates with majors in mathematics are becoming elementary or secondary school teachers. Finally, when one discusses mathematical education in the Southern Negro schools, there are many other facts that may be taken as assumptions. Some of these are the un-availability of a wide range of supplementary teaching materials, in each school, inadequate libraries, isolation from the main stream of educational activities, and the general attitudes of some supervisors and administrators. The details for all of these would take us too far from our main objective, but we shall refer to them in passing as necessary in the development of our talk.

The main objective of our talk is to review the mathematics program of the Southern Negro school as we see it and feel it relative to our subject, of course. Although many deterrents may be cited, we shall limit our talk to discussion of the children and to people who have direct contact with the children.

The main people to be discussed are the child, the parent, the mathematics teacher, the administrator, and groups of these. We shall focus our attention primarily on the teacher and the parent.

First, let's take a look at the teacher. The Negro mathematics teacher in the South - good or bad - is in a rather enviable position. Often without any real effort of his own, he is held in high esteem by his peers, in many instances by his supervisors, and by the members of the community in which he works. This esteem is generated in his peers by their recalling certain difficulties encountered while in college mathematics courses and by certain problems in their daily routine of pupil accounting and other menial mathematical tasks. The high esteem held by his supervisors is proved by the many cases where mathematics teachers (particularly men) have been moved up into administrative positions. The people in the community hold the mathematics teachers in high esteem because of their mathematical experiences in school, their experiences in filling certain forms such as income tax forms, their inability to perform on the mathematics sections of examinations, such as the Civil Service Examination or college entrance examinations, and, strange as it might seem, their sincere concern for the mathematical training of their children. Children also have their own notions about teachers and courses. First, students classify courses as hard or easy. They generally classify mathematics as a science; they classify science as hard and they classify mathematics as the hardest of all the sciences. Since Mrs. X is a mathematics teacher, they reason, Mrs. X must be smart. Even in the educationally deprived group a reputation of being smart commands a certain respect!

From a subject matter point of view many mathematics teachers have not earned this status, yet they enjoy it. This very unmerited esteem leads to our first deterrent. This mathematics teacher reasons that a man with this esteem must never let it be known that he is ignorant of certain basic facts in the subject matter that he is teaching. To hide this ignorance he must devise a systematic method for meeting every situation. In the classroom he hides behind requiring each student to execute certain problems by some prescribed technique. He discourages creativity and imagination. He will not entertain questions on the origin of ideas and the why of things. He will consider only those exercises that he mastered in previous years. He generally omits the verbal problems, and he rejects the adoption of new materials or techniques and then rationalizes his deficiencies through complaints of the school's structure or the substandard characteristics of his students. In his relationships with other faculty members, no one presents any real threat except other mathematics teachers. To hide some of his deficiencies when in the company of other mathematics teachers he either ruthlessly avoids discussing subject matter or he identifies himself as an algebra teacher or with some specific subject matter. This denies the existence of any common teaching problems. He receives enthusiastically any discussion on team teaching, but carefully avoids discussing instructional problems with a team of teachers.

If our observations are correct, this teacher is the one who has most of the classroom disciplinary problems and the attendance rates for his classes are relatively low. This very teacher will also do nothing about returning to school for additional training, even though a substantial stipend may be available. But, let's give credit where credit is due. He is not stupid. He probably will not have an experience like that of a mathematics teacher who happened to have her supervisor walk in while one of her better students was presenting his proof for a certain plane geometry theorem. About half way through the proof he "lost" the teacher. When he finished the teacher thought she had to make a decision on his work. She made a decision and unfortunately she was wrong. What we are talking about here is the major problem of esteem versus preparation. Now, who is to blame? To relate properly to this question one could divide all Negro mathematics teachers (in the South) into two sets. In one set we place those teachers whose transcripts indicate sufficient mathematical training and who are otherwise fully qualified to teach mathematics. To entertain any detailed analysis of these teachers during this talk would be a trifle out of place. In the other set, we place the other school mathematics teachers who have not received from any source sufficient mathematical training. It seems that the latter set is more numerous. Some of the contributing factors are unwise planning in the past, the urgent need for filling positions immediately, the filling of positions to satisfy pressure groups, political pull, and the general feeling of some personnel directors that anybody can teach mathematics.

This factor of preparation isolated from all others dooms many teachers to at least nine months of frustration a year. How do they survive? First in the classroom the Negro mathematics teacher is almost an absolute monarch. For the most part, even in the large districts, he is free to operate at will. He can elect to teach mathematics or simply kill time. He can use the text, notes, aids, etc. or he can use nothing. He can cover the outline (if he has one) or he can cover twenty pages. He is free. If he elects to do nothing, as he does in many instances, he can assign grades of A's and B's to the children of the bigwigs of the community, give C's and D's to the other children who attend class regularly and F's to those who failed to meet the minimum daily attendance requirements. Then everybody is happy. If he fails to do this, many misguided parents will question the poor grade but never, and I do mean never, question how much material was learned by the child. When questions about grades are raised, usually the teacher produces strong written evidence. There have been some cases where steadfast teachers have been overruled by administrators to the satisfaction of the parent. Hence, the desire of the teacher to resist the lowering of standards has been blocked by a force beyond the control of the teacher. If a question relative to the grade issued to a member of the educationally deprived groups is raised, the poorly prepared teacher can usually rationalize the grade through such escapes as absences or disciplinary problems.

What about the very good teacher? In more than a few cases a teacher with outstanding ability will come along. He knows the subject matter to be taught. He knows that the learner needs help. He offers this help. He supplements this help with encouragement. He is a friend to the learner, and lends sympathetic understanding to the learner's failure to learn. He then dreams and tries new approaches. This teacher is good. The administrator knows this and extrapolates: This man would probably be good in anything he does - so he should chair important committees, he could assume positions of responsibility and soon he does. But he has limited energy, and therefore must draw from his classroom energy. Where once he used his imagination to promote learning, he now uses his imagination to promote school activities outside the classroom. One might ask, "why doesn't he resist?" To resist would be to fail to cooperate. His yearly rating might fall. He might even be declared undesirable. Furthermore, it is through these activities and not through excellence in teaching performance that he gets his commendations and special privileges. Where is his supervisor? In many cases he has a supervisor in name only. In other cases contact with the supervisor without the expressed permission of the principal is frowned upon. At any rate, the supervisor knows this and conducts himself accordingly.

Fortunately, most of the mathematics teachers we know do elect to cover some mathematical material in the school classroom. If they once were complacent with their end product, this complacency has been overruled by their sensitivity to the new opportunities for their students. In my opinion, this sensitivity might be used to build a foundation for an improvement program.

Before I turn from this brief discussion of the Southern Negro mathematics teacher, it seems pertinent to mention the obvious decline in the prestige of mathematics teachers (particularly males), if they remain in the classroom for a period of many years. We think that this is due to poor judgment which specifies that if he is worth a grain of salt he should move up. Even once I was told by one of my co-workers in a secondary school, "Young man you have been in the classroom four years now, five is enough. You go get some education and get a principal's job." This lady meant well. In fact, I am inclined to believe that she had overestimated my teaching performance. What is more important, however, is that she believed, as many others believe, that if a mathematics teacher is really good, he should not be in the classroom - he is wasting his time. Why earn $5,000 annually teaching school mathematics when opportunities to earn much more money are available?

Now let's take a look at the parent.

In order to get a good cross section of the parents of low achievers, we think it necessary to classify the parents into two sub-groups. We shall define group A as that of families whose incomes can be equated to these middle-class Americans and we shall place the socio-economic deprived families in group B.

Out of group A, we find many children whose projected vocations are unrealistically high. These very high vocational goals are set up by the parents and the youngster is charged with the responsibility of living up to these unrealistic dreams. It seems that these dreams are built on the parents' ability to support their child through college with no recognition of the child's achievement level or mental capacity. Generally, the communities of Southern Negroes with (moderate) incomes are so small that the existing selective social group in each community includes the teachers, administrators, and other parents from group A. Moreover, the thinking is so narrow that any negative statement relative to performance of a child from group A puts the teacher's social position in jeopardy. Hence, in many instances the cynical upper-status self-satisfied Negro teacher will conduct himself in such a way as to allow these parents to enjoy the illusion of having a very bright child. So what about mathematics? Well, these parents are educated or have guidance resources and they do know what courses are required to attain a certain goal. This is good because it gets the child into the so-called heavy courses in the curriculum. After getting the child into the course, however, no provisions are made for the child to meet the needs of the course. No one has told the parent that you don't learn mathematics in the classroom. So they continue to live through the child. They want the best for him. They fill every moment of his time with assigned duties. They expect him to perform numerous school activities, church activities, scouting, dance lessons, drama lessons, etc. In many cases the child becomes frustrated because of these excessive demands placed on him by his parents.

We believe one other point worth mentioning is that the parents in group A often tend to patronize their children. They pay for success and their children realize this. The child has measured the accepted get-by level. He attains this level and never feels the need for getting ahead. Therefore, if he can get a grade of B or A for mediocre performance why should he extend himself, why should he read, why should he subject himself to the hard work of learning. So his total performance is geared to meet the needs of the day and not on preparation for tomorrow. This attitude inevitably leads to low achievement.

Parents of the socio-economic deprived groups represent by far the larger of the two groups, and they present quite a different picture. These parents might be considered as typical throughout the South.

For many years the Negro in the South has been trained by whites and other Negroes that he has a precisely defined place in society relative to economic and occupational opportunities. This lesson was learned well, for even today, it is not strange to hear people from other social groups and some Negroes refer to certain occupations as a white man's job or a colored man's job. When a Negro parent with these deep-seated notions projects vocational opportunities for his child, in his mind he is faced with making a choice for his child within the system as he sees it or preparing his child to escape from that system. In too many instances his choice is motivated by his occupation and other forces in his environment. That is, in some instances he can get aid for his child's education if the vocational choice is correct. In other cases the local educational institutions are tooled for training Negroes to exist inside the local social occupational structure. If the parent has limited financial resources, he has no choice but to maintain his child in this situation. The single fact that the parent has in mind the aforementioned grouping relative to occupational opportunities serves as a deterrent to the educational development of the child even though educational opportunities may be available in the community. Many of the parents in group B are characterized by rather strange reasonings relative to their child. They reason that "I made it on what I have, so you can make it on the same or very little more." Or they reason that, "I made it through hard work, strife and pain, so in order for you to appreciate anything you must be subjected to some of these hardships." They don't realize that getting an education today is identified with that very same hard work, strife, and pain.

In entering a discussion of the child, segregation is an extremely important factor in the development of the low achievers. I do not wish to spend time discussing the mixing of races in the classroom, although I feel that this is one of the better ways for motivating excellence in performance in all groups. Rather, I would emphasize segregation as it occurs in the existing Negro school in the South. We pointed out in our discussion of the teacher that there is a form of segregation among the mathematics teachers themselves. Another form of segregation is manifested by the lack of communication between some teachers and the child from the lower socio-economic groups due to the teacher's negative image of the underprivileged child. Of all the children in the schools the underprivileged child is affected more by segregation than any other child. Here's how this is done. It is a known fact that students from the underprivileged groups generally do not seek knowledge for the sake of knowledge and that there is a high correlation between their aspirations and their opportunities. Furthermore, this child has little or no chance of getting wise and thorough guidance information at home, at church, or from any source other than the school. Hence, the whole of his non-religious educational development is centered in the school. I remind you that we are talking about children who don't know how to ask for information and who are also aware of the negative attitudes of some of the teachers and counselors toward them.

The responsibility for the issuance of guidance information to these children falls squarely on the shoulders of the counselors. The counselors underestimate the possibility of the economically underprivileged child's going to college. The parent does not know and hence the child is often guided into some non-college preparatory direction and is virtually forgotten except in head counts. This shifting is extremely important because it brands this underprivileged child with a stamp of inferiority and hopelessness. Moreover, this shifting is going to subject this child to the poorest classroom teaching available in his school. The best classroom teaching is reserved for the known college bound student because the school is sensitive to the performance of its graduates in colleges or universities.

On the other hand, the student from group A is encouraged beyond his achievement level. He is sought out to perform in many activities and eventually becomes well known throughout the school. If honor sections are established, this child will probably be included in one or more of these sections. He develops a feeling of superiority. In connection with this, I have never seen an honors section in anything except teams of athletes with more than four or five students from the lower socio-economic groups.

Further examples of segregation are evident in the parent-teacher relationships. The parent from group B is too often made to feel he is substandard and that he is privileged to have the school admit his child. Over the years certain parents of the lower socio-economic groups have learned to literally hate teachers and consider the school as a baby sitting operation for about seven hours a day. The child is obliged not to learn under these conditions.

Isolation is an important factor in the development of low achievers in all Negro schools in the South. Being outside of the mainstream of educational activities causes the establishment of diversified norms at individual schools. These norms generally turn out to be far below the national norms; hence, we find many valedictorians unable to survive one year of college work.

Another deterrent is the obvious lack of clearly defined purpose for a course or a topic, particularly in the mind of the student. The average Negro student is aware of the applied mathematics necessary to answer simple questions like how much and how many, and he is willing to work and master those basic skills necessary to meet these needs. Unfortunately, many of these students are led to believe that these skills represent the whole of mathematics. The interested teacher, on the other hand, has in mind helping the student to become reasonably aware of what mathematics is, what it does, why it does it, and how it does it. Hence the teacher tries to create a learning atmosphere in the absence of unity of minds. To get his job done, the teacher relies heavily on the text. Too often, the text materials and illustrations are drawn from middle class American life and are completely irrelevant to the concerns of the student. The nonimaginative teacher adheres to this text material and emphasizes the correct answer. Two things are out of place here. First, the inability of some students to work through to the correct answer is interpreted as ignorance, and second, the attainment of isolated results without review or generalizations leaves the student with very little fortification for future problems. Out of experiences like these, the low achievers begin to think that every mathematics course is an entity in itself. Moreover, the student never learns to transfer knowledge gained in mathematics courses to other courses. Hence the physics teacher, the economics teacher and many other teachers have to reteach the mathematics necessary for their course.

Our focus now turns to the administrators. We must be mindful that many Negro school principals in the South have allowed themselves to be put into positions where they must make so many adjustments in the forces stabilizing their position that they don't have time to really run their schools. Among these principals are those who are put on the job by the local power structure because of the "their" ability to define and maintain racial lines. Also among these principals are those who let themselves get so tied up in the personal lives of some of their faculty members that they must be biased in their dealings with the faculty. This inevitably leads to poor teaching performance because of the high correlation between job success and personal satisfaction in work and working conditions. Another deterrent to the principal's establishment of a learning atmosphere is his sensitivity to his social position and his fear of being unpopular in the local social structure. That is, what he does at school affects his life away from school and as much as he would like to correct certain ills in the total learning environment, he refrains because influential people are involved.

Before we summarize some main line deterrents, we think that we should indicate the lack of classroom time as one of the main deterrents in school education. Consider this partial list of activities consuming school time: fixing enrollment cards, Columbus Day Program, Thanksgiving Day Program, Christmas Program, committee meetings, movies of all sorts, physical fitness test, P. T. A. meetings, Education Week Programs, fixing bulletin boards, club programs, interruptions over speaker system, Easter Programs, May Festivals, pep rallies, coffee breaks, cigarette break, conferences, faculty frolic, talent shows, and many more programs and money raising activities.

We now list ten main line deterrents that we think contribute in varying degrees to the development of low achievers. Two other reasons for listing these deterrents are (1) that many of these deterrents probably could not be documented and (2) if these deterrents are major, it is time that we spell them out and meet the problems head on. These main line deterrents are:

1. Because of the basic desire of many members of the power structure in the South to keep Negroes in their so-called place, no real emphasis is placed on how much material is being learned by the child;
2. The standards of many schools are tuned to the desires of an uninformed public;
3. There is a war of jealousy and bigotry going on between professional staff members;
4. Too much classroom time is used in meeting the demands of the community that the school become a show place;
5. Counselors underestimate the vocational opportunities of the students;
6. Many teachers use poor I. Q. scores as an excuse for not teaching rather than for planning a base line for teaching;
7. Many teachers have negative attitudes toward the child from lower socio-economic groups;
8. Many poorly trained or disinterested teachers are in key positions and use their positions to block all attempts at an improvement program;
9. Many teachers are lacking in basic knowledge in the subject matter; and
10. Many teachers lack the facility to communicate with the child from the lower socio-economic groups.

In connection with this last deterrent it seems that the institutions training teachers who will work with children from the lower socio-economic group may have to look into communication techniques and revise their teacher training programs.

Lest I be misunderstood, I should clarify my position, I am not saying that the deterrents mentioned are all applicable only to mathematics or to Negro schools nor am I saying the existing situations are so deplorable until any attempts at an improvement program would be in vain. I am saying the situation is generally bad but out of this bad situation comes the cries of thousands of teachers who want to teach and are looking for help. Also, in this situation I see a vast reservoir of untapped brainpower ready to be released by some operation. My contention here is that if you are contemplating a bootstrap operation you should know something about the strap.

Before I close, may I summarize my feelings toward what you are contemplating doing. I admonish you to remember that we are talking about thousands of teachers and millions of children. We are talking about one of the highest yielding investments possible. We are talking about adding to the wealth of the nation. We are talking about the mathematical literacy of millions of Americans and the never-to-be overestimated by-product of thought processes one develops through the active learning of mathematics. I do hope that you decide to move ahead with this project. Nothing could be more worthwhile.

DISCUSSION OF CLARKSON'S PAPER

Much of the discussion turned on the problem of teacher education particularly the in-service training of teachers with modest backgrounds in mathematics. One difficulty is the reluctance of poorly-prepared teachers to expose themselves to retraining. Another difficulty is the poor preparation of those in charge of teacher-training programs.

The conclusion emerged that any program for retraining teachers on a large scale must be carefully considered and must involve all of the teachers in a school.

Clarkson recommended that teachers be shown specific techniques for giving assignments and offering heuristic hints to students. He also suggested material be provided for counseling students to help justify the mathematics being taught.

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SUMMARY OF GENERAL DISCUSSION

I. Mental Development

The transition from one stage of thought to the next is exceedingly important both in teaching children and in research on the development of thought processes. The movement from concrete to gradually greater degrees of abstractness of materials must be made slowly. Another problem for research endeavor in addition to that of transition between mental stages is to discover the habits of thought which make mathematics instruction possible.

II. Description of Low Achievers

Low achievers may be classified into three groups:

1. Those children who have intellectual deficiencies.
2. Those children who have cultural deficiencies.
3. Those children who have both intellectual and cultural deficiencies.

Perhaps a qualitatively different group are the children who are neither culturally nor intellectually deficient but who are slower learning. Although these children may not be less intelligent than the average group, they take longer to process information.

It is important to consider these differences among low-achieving pupils in teaching, both for individualizing the curriculum and for developing self-instructional materials.

In motivating low achievers classified as culturally deficient, two factors of crucial significance are (1) the need to develop "docility" in order for school learning to take place, and (2) the need for flexibility in approach to teaching depending upon the value systems of the homes from which these children come.

III. Action Programs

In discussing possible programs for compensating for the deficiencies with which culturally deprived children reach school, several suggestions were made. The first was mass instruction at the pre-school level, which would involve a large investment of money in new facilities. An argument raised against such a program was that the pre-school skills which could hopefully be developed are difficult to handle in group settings.

A second suggestion was to improve existing school programs, more fully utilizing existing resources. This could be achieved by extending the school day, using school facilities during summers for nursery and pre-kindergarten instruction, and by establishing state Peace Corps groups to staff these schools.

A third suggestion was for supporting an omnibus education bill which would include both retraining of unemployed adults and preparing culturally deprived children for school. It was pointed out that thought has been given to such a bill by the Office of Education, but it is difficult to get such an omnibus bill passed by Congress. In this context, the point was made that the experiences of the Ford Foundation and the Great Cities Program for School Improvement have demonstrated the value of a "total approach."

The question of whether to involve parents in such programs for culturally deprived children was raised. Arguments against involving parents included the hostility of schools to lower class parents and the difficulty of getting these parents to come to the schools. Although some conference members felt that considerable mathematics and reading instruction is done at home, others thought that it is not typical for parents to help their children with school work and that materials should be devised so that instruction and learning can take place almost entirely in school.

In considering the training of teachers, it was suggested that any program for retraining teachers in the new mathematics curricula must be well thought through and must involve all the teachers in a school. It was suggested, further, that teachers be provided with material for counselling students to justify their studying mathematics.

IV. Demonstration and Research Programs

It was suggested that a group of users of mathematics be assembled to point out the kinds of mathematical problems encountered in their particular fields. A group of mathematicians could then determine what mathematical principles are needed to solve such problems.

A demonstration program was suggested in which a school in a culturally depressed area would be staffed with people engaged in research on curriculum and teaching, as well as observing students, to find out what would work if money were no object.

These last two suggestions raised in the discussions are related to recommendations made by the small groups in their individual reports.

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James Hicks

CURRICULA AND PROBLEMS OF TEACHING MATHEMATICS IN A VOCATIONAL HIGH SCHOOL

Well, first of all, I'd like to tell you a little bit about my school, Dunbar Vocational High School...Dunbar Vocational High School was not always a separate high school nor was it always considered to be a high school.....was a regular academic high school. It was used as a collection point for undesirable students..that is, students who could not be handled by regular procedures; and the philosophy, I believe, at that time was "if a boy can't learn in an academic school then send him to Dunbar and teach him a trade." So, this was the start of our school. It became a collection point for undesirables. It grew in size and gradually the faculty was enlarged and it was finally decided by those in authority to make Dunbar a separate school and to call it a "Trade School." At this particular stage of Dunbar's development all mathematics was related, if any math was taught, directly to the shop. There was little or no consideration given for such subjects as algebra and geometry. Now, the second thing about my school is that Dunbar is located in an area here about two miles south of where we are now, and I believe that you would call this a culturally deprived area. Now, the area is this type of area--I think we could divide it into three or four parts--we have one part of the area which has been cleared under urban renewal and on this particular land they have built a long block after block of high-rise, public housing. The second part of this area would be the area that is cleared, that is the land that is now vacant has been cleared under urban renewal and now lies vacant. Then, we have the housing in this area, remaining housing, which is considered to be substandard and, finally, we have the factories around our area. So, this is the area from which we draw our students and the comments that I am going to make apply to slow-learners. All of our students are not, of course, slow-learners and since the days (Dunbar is about 22 years old now) when Dunbar first became independent, we have made some strides and we have attempted to dispel the notion of sending a youngster to school merely to learn a trade. At the high school level, we are apt to become more aware of the results of a lack of early mental development rather than its cause. In attempting to teach abstract concepts in mathematics to slow-learners, we are almost immediately made aware of his difficulties in communication, both verbal and written, his slow reading comprehension, and his short span of attention. All too often, his response in class is limited to a muffled grunt accompanied by the movement of the lip. This recalls--not too long ago, in a class I was visiting, there was an oral session and the class was reciting in unison and, sitting in the back of the room, I observed that it was somewhat like a symphony. Some people were using their voices to sort of add body to the general comments, some would answer the question, and others would sort of grunt along. It gave a remarkable effect in the front of the room I imagine but, in the back of the room, it didn't sound too well. But, all too often we find that this is the situation we are confronted with. If there is not some form of homogeneous grouping the slow-learner's attention is likely to wander and disciplinary difficulties are thus created. In view of this fact, it seems that the curriculum should be based on some form of homogeneous grouping. The most logical basis would be reading comprehension and mathematical achievement. This would facilitate the selection of suitable textbooks and supplementary materials. Now, when we are dealing with the slow-learner who has the communications problems, many of the books that he is supplied with are books that are geared to the, what we call, average American student. They are books that are pretty to look at, well designed on the cover, using many colors, but books that have language that the slow-learner is not likely to understand. The slow-learner will not be inclined to read too carefully when he finds that the language used in the books is a little too difficult for him to understand. Grouping might also be a stimulant to the slow-learner for more active classroom participation, thus providing a means of evaluating, to a degree, the effectiveness of the program. General considerations would dictate that the curriculum neither be at the same rate nor depth as the program for regular students. Provision should be made for varying subject time allocations with the usual concept of the daily 45, 60, or 90 minute periods, subject to some alteration as the need arises. I think, also, that we have become assuaged to the idea of a regular period, meeting at the same time every day, so many days a week. We also always meet for 40 minutes, 45 minutes, or 60 minutes. In many cases it would probably be more beneficial if we were able to give more time to these under-privileged youngsters. Noting that they already have a learning lag, the extra time would be used to fill this lag, to bring them closer to "normal" achievement levels. Cumulative drill and review must be an integral part of the program for all too frequently the slow-learner loses his mastery of previous concepts and begins to exhibit a natural reluctance to accept new and perhaps more confusing ideas. Quite frequently, we find that the slow-learner is slow to grasp an idea or a concept and, having grasped this concept, he finds that just as he is about to utilize his knowledge of the concept, the teacher has moved on to the next subject and he is behind once again and his problem is just repeated. His problem--he hasn't been helped and, eventually, in trying to learn the new concept he falls behind again and this becomes a chain. Moreover, the curriculum should not emphasize unrelated tricks and manipulative skills, but rather it should pose a challenge that the learner is capable of accepting. Now, in many classes, we find that the discussion of geometry and theorems in geometry, of ideas in algebra...these do pose a challenge, but it is quite possible that the slow-learner would never, even with eight years in high school, be able to accept the challenge posed by these problems. Therefore, we are wasting his time when we are teaching subject matter that he cannot possibly comprehend at that particular point. In the vocational high school, there is a duality between a shop and its related mathematics courses. The shop supplies a practical laboratory for the accumulation of manual skills. Mathematics then becomes a practical and useful tool when used to extend and to complement these manual skills outside the shop. I think that, in this group, we must remember that the motivation for going to a vocational school is to gain competence in some trade area and the eventual goal is to earn money. This is a very important consideration--to earn money--and this is above all other educational considerations for members of the slow-learners' group. Now, in this particular group, if we cannot substitute new goals for his old goals, then it would seem to me that we are going to have to cater to his goal. This practical approach to mathematical curricula often finds acceptance from the slow-learner for it is based on educational goals which are meaningful to the student. Now, among our various problems in the vocational high school curriculum we are two things--first of all, we are a vocational school which means we are obligated to teach the student a trade. We are compensated under the Smith-Hughes and Smith Lever Acts and we must have them in shop for so many hours a day. Opposed to this, inasmuch as we are a high school, there is pressure from the local school of accreditation organizations to teach certain subjects. Therefore, the time we are able to spend in mathematics is extremely limited. There is no time for such things as mathematical clubs, extra activities that might involve mathematics. The student, and the school, is mainly involved in training students for vocational competence. Another problem is our attendance problem and drop-outs. Our freshmen that we receive have very low scores of achievements and our curriculum is not geared to teach them. Furthermore, our teachers are, possibly, not properly trained to teach at this particular level. Other problems are textbook shortages. Not only are our textbooks old and the ideas no longer useful, but the new textbooks are more costly, they've risen in cost, and we are not allotted an amount of money which allows us to purchase textbooks which would be suitable for training the low-achieving youngster. Last, the home environment of our pupils, the low-achievers--there is not encouragement to learn. There is no male influence in the home and, consequently, the low-achiever can do as he chooses. This means that this will result--this is going to result--in more days out of school, more youngsters being sent to special schools, for non-conformity to school rules and, eventually, more of our students are going to drop out because they are just not successful.

Robert M. Gagné

A STUDY OF KNOWLEDGES AND SKILLS COMMON TO HIGH-DEMAND TECHNICAL JOBS

I am happy that you are addressing yourselves to this rather large problem of what to do about the lower half of the school population. Concern for these people is long overdue. And I hope that you are able to come up with some helpful ideas about the education of the under-privileged, the slow-learners, the forgotten people of education.

I want to tell you about a project in which we are engaged at the present time. It is designed to identify the generalizable knowledges and skills that are common to certain important vocations. I need to say that we haven't been at this very long, so that I really don't have the results of the study to report to you. I believe that these findings, when they do appear, will be of interest and importance to you in curriculum planning. But at the moment I will be able to give you only some background information, which I hope will be helpful in furthering the purposes which you are trying to meet in this meeting.

The purpose of the project which I want to describe to you is to search for leads as to what may be the requirements of general education, for those who enter vocations. I think you know that in many large and comprehensive high schools and also in vocational schools students take courses which are pretty much like the courses for the college-bound, or perhaps watered-down versions of these courses. The important question is to try to determine what should be the kind of curriculum that is offered to people who are going to go into vocations. We hope that the study in which we are engaged will throw some light upon this question. Its purpose is not to develop such a curriculum; it is simply to get the information which will make it possible to make sensible decisions about what such a curriculum should be, in order to best equip these young people for the world of work and for any vocation which they wish to enter.

I would like to describe what we're trying to do in terms of certain steps. The first of these is finding what jobs we're going to be concerned with. In other words--Selection of a job sample. Now the question that we're asking here is fairly simple, but it isn't too simple to find a good answer. The question is: What are the jobs that are going to provide the greatest number of opportunities in the next ten or fifteen years? You might think that one would come out with an answer which emphasized certain types of activities which are highly visible nowadays, such as those having to do with the use of computers. But we do have to distinguish between those vocations which have a high growth rate, on the one hand, and those vocations which are going to provide a high absolute number of jobs over the next 10 or 15 years, on the other.

The two accompanying tables are intended to provide you with some basic information about vocational courses and about vocations. Table 1 shows the number of schools in the United States which offer vocational courses having the titles shown, the latter listed in order of frequency.

TABLE 1

NUMBER OF SCHOOLS IN THE UNITED STATES OFFERING VARIOUS VOCATIONAL COURSES

(Secondary School Level)

Course	No. of Schools
1. Auto Mechanics	731
2. Machine Shop	616
3. Carpentry	351
4. Printing	311
5. Drafting	246
6. Electrical Trades-Industrial Elective	197
7. Welding	145
8. Cabinet Making	142
9. Bricklaying	137
10. Radio and Television Service	126
11. Sheet Metal Work	120
12. Cosmetology	94
13. Food Trades	93
14. Building Trades (General)	92
15. Beauty Culture	90
16. Auto Body and Fender Repair	88
17. Mill and Cabinet Making	84
18. Electrical Wiring	74
19. Electronics, Basic and Industrial	72
20. Art, Commercial, and Advertising	61
21. Metal Trades (General Metals)	59
22. Dressmaking	54
23. Practical Nursing	54
24. Tailoring	42
25. Plumbing, Pipe-fitting and Steam-fitting	37
26. Refrigeration and Air Conditioning	35
27. Aircraft Mechanics	31
28. Building Construction Mechanics	27
29. Painting and Decorating	27
30. Furniture Upholstery and Repair	23

Table 2 contains another listing which is perhaps even more interesting for your purposes. It is a ranking of the kinds of vocations, job families if you want, which are going to be most frequent in opportunity over the next 10 years. Now where did we get this? We asked experts in the Department of Labor who work on these forecasts, to generate this table for us. We tried, of course, to get the figures ourselves from various sources, but the data from these sources are incomplete as I am sure you realize. The Department of Labor appears to have the best sources for this kind of information. They ranked the jobs as shown in the table. Here we find secretary-typists on the top of the list. These are an interesting set of occupations, and I don't know if you can detect any trends here or not. It might appear that the service occupations seem to be getting a lot of weight. Only the top 30 vocations are shown in the table.

TABLE 2

RANKING OF EMPLOYMENT OPPORTUNITIES IN OCCUPATIONS FOR WHICH VOCATIONAL EDUCATION IS RELEVANT

Rank	Occupation	Opportunities
1	Secretary, Typist	Many
2	Salesman	Many
3	Waiter, Waitress	Many
4	Bookkeeper	Many
5	Truck Driver	Many
6	Machinist	Many
7	Sewing Machine Operator; Tailor	Many
8	Auto and Diesel Mechanic	Many
9	Real Estate Salesman	Many
10	Beauty Operator	Many
11	Carpenter	Many
12	Insurance Agent	Moderate
13	Practical Nurse	Moderate
14	Cook and Chef	Moderate
15	Painter	Moderate
16	Electrician	Moderate
17	Plumber, Pipefitter	Moderate
18	Bank Teller	Moderate
19	Welder	Moderate
20	Draftsman	Moderate
21	Postal Clerk	Moderate
22	Stationary Engineer and Fireman	Moderate
23	Appliance Serviceman	Moderate
24	Mail Carrier	Moderate
25	Policeman	Moderate
26	Bricklayer	Moderate
27	Electronic and Electric Assembler	Moderate
28	Fireman	Moderate
29	Operating Engineer	Moderate
30	Instrument Technician	Moderate

So much for the first step. Our next step is to make job descriptions for each of these jobs that we select as being representative of what is going to be available. Now these are rather detailed kinds of statements that we are seeking. They do not exist in print. The summaries in such places as the Dictionary of Occupational Titles, the Armed Forces Civilian Job Standards, and other printed descriptions of that sort, while they are helpful, are not at all as detailed as we need. We need to get down to a very detailed level to find out what it is that these people do on the job, because we are concerned with human behavior and its measurement.

Then there is the stage which I will call translating performance descriptions into test items. These are not aptitude tests; they are job sample tests. They are designed to find out whether the individual can, in fact, perform the kinds of activities that are required by the job. If, for example, we were to find that a job has to do with reading meters of the sort which have a needle and a scale, and require interpolation between scale marks, then that might be used to represent a task which could be measured.

What we're going to do is to administer tests which are constructed to represent the kinds of things which people do on the job to a fairly large group of vocational students. This group will include those in high school as well as those in more advanced technical schools or junior colleges.

Out of this administration we will get a great mass of data about how people perform on these tests, and the next step is to find our what is general about their performances. Now there are various ways of doing this. It has been done in the past in many cases by making good, informed judgments. One looks at the kinds of specific things that people do on the job, and says, "I think that this individual needs to have a general knowledge of such and such." Now this is not an unreasonable method, but we are not sure that it is the best method. Our approach in this case is to do a factor analysis of these test scores. Factor analysis is a well known technique which is based on mathematical reasoning. It is a means of finding, in quantitative terms, the degree of communality that exists between performances when they are measured. And this is exactly what we are looking for. We're trying to find out what is common among these scores of the specific items that are based on job descriptions. The basic technique is to correlate these scores with each other, obtaining first a set of product-moment correlations. Factor analysis is a technique of looking at a matrix of correlations and finding out what common factors may be present in them.

We don't know what kinds of general factors we're going to find by this method. One can make certain guesses here, but we're not sure of these as yet. For example, many jobs involve factors of personal safety; maybe we will find something in common among many of these jobs which pertains to the principles of personal safety. Many jobs also involve sanitation principles. There are also many jobs that involve drawing sketches, such as a carpenter may draw, or an appliance installer. This isn't mechanical drawing exactly, but it is being able to make a rough sketch, perhaps in three dimensions. Communication is obviously involved in many jobs, and this may be a factor which we find in common among them. What is there in mathematics? Again I can only guess. I think we may find some very interesting mathematical things in common to these jobs. Obviously many people in such jobs as these have to deal with proportions and ratios. I would think that many of them have to deal with the manipulation of positive integers. We propose to find out by means of this empirical method.

The final step, once we have identified the common factors, the common requirements of jobs, is to look at them in terms of what they mean for the kinds of education that might be given to these students. In other words, to draw the implications which these generalizable skills and knowledges may have for design of the curriculum. We are intending to draw these implications and see where they lead us. We hope that inferences can be made about the kind of curriculum which would be most useful in vocational education.

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PANEL DISCUSSION

H. Pollak:

I came to this meeting in the hope of learning something about the whole problem. I've just started to do that, so I certainly am very far from qualifying as an expert, at least until I get home. So I really don't feel that I have very much to contribute, except maybe a couple of observations. I observe that a very large proportion of what I've been hearing is really very relevant to the general problem of education. In particular, many of the things we have heard about motivation, and many other aspects of the problem of teaching, seem to me to apply not only to the problem of the under-achievers, but also to the education of all students. So I'm encouraged to try this argument in reverse, and hope that some general educational comments about applications of mathematics might apply, in particular, to the under-achievers.

I think one of the topics in mathematics that we should examine to see whether it isn't a good one for our purposes is probability and statistics. This is something that was discussed a good deal last summer at the Cambridge Conference, particularly in terms of very early presentation, probably even in the elementary schools. We saw here an opportunity for very concrete mathematical experience. We were thinking in terms of kids getting down on their hands and knees and throwing thumbtacks, keeping records of what was happening, and beginning to learn something from them. An activity of this kind provides a sneaky way of practicing arithmetic in a situation which might be interesting, where you didn't know you were doing it; a feeling for sums and averages and how they behave; a way of learning how important it is to keep track of the exact conditions under which you are making your experiment. For, it turns out that if you throw thumbtacks in different ways, you may get very different answers. Perhaps you don't expect this, but this is what happens, and it is a good lesson for life. Some of us have had the feeling for a long time that the earlier you start on probability and statistics, the better it is. If you wait to start in college, you may have kept many kids from ever learning the subject, because the kind of thinking is so different. Here also is an opportunity for motivating students that you might be able to motivate with very little else. If you give them an opportunity to have some better feeling for odds and games of chance than they might otherwise have, they might actually be willing to work. It wouldn't look quite like the arithmetic that they have been exposed to all along, and I would certainly recommend it. Here, incidentally, is one of the topics which has great vocational opportunities. Now speaking at another level, besides those vocational opportunities, one of the greatest lacks in the education of our engineers and of technical aides is the lack of probability and statistics. This is an enormously important field in all kinds of applications, and I think it offers us a real opportunity.

To show how far mathematical education of people like engineering assistants is going, let me comment on a recent visit by an official of a Radio Engineering Institute who was looking for some help in writing courses. These are mostly mail order, although there are also some full-time students. He had a list of about eighteen advanced mathematics courses. He didn't want to use standard textbooks, because the courses are specifically for students interested in electronics. Thus, all the examples and the motivation were from that field. The subjects included not only the whole calculus sequence but ordinary and partial differential equations, matrices, complex variables, probability, etc. It was quite an impressive list. All this is for a person who is not really going to college, but to a technical institute. It's an interesting trend.

I should mention another subject that came up frequently at the Cambridge Conference. We felt that it was very good mathematics, and very good applications of mathematics, and also very good pedagogy, occasionally to teach kids in terms of "Here is a situation, think about it," rather than "Here is a problem, solve it." We always give students explicit problems to do, and we always expect explicit answers. It would be very good if we could say to them occasionally, "Look, here's a situation, whether it's a mathematical one, a real life one, or what not. Try to get some understanding of it. What do you suppose is going on?" I think there are many opportunities for this throughout the schools, and it might be something very appealing to kids because it's not the usual way of attacking mathematics. Such an attack, of course, is also excellent applied mathematics because in applications you frequently don't have a particular, well-defined, clear-cut problem all given to you on a silver platter. You fortunately (or unfortunately, depending on your point of view) have to find the right problem first, the right question to ask; then you have to see how to attack it. It would be very good if our education did a little more of this.

We now come back, of course, to the main problem that has been discussed. You can't very well expect teachers to do this in the classroom if it's never been done to them. It was never done to me until I was in graduate school. I'm sure that many people never see this done to them at all, and yet if we are going to have this kind of teaching, we are going to have to break the cycle somewhere.

P. Suppes:

I'd like to make about three comments, mainly directed towards the problem of elementary school mathematics for the underachievers. I speak with considerable hesitancy. I don't feel that I have any firm things to say about it. I do have a sense from listening to the discussion this morning and references we've heard that nobody else has too much to say about it yet either. The actual problem of what to do with the curriculum is a technical problem that doesn't seem to have been worked on very much as yet. I speak particularly of the technical problem of what to do with the elementary school mathematics curriculum for the lower achievers. It seems to me that the serious question we face is how should we change the curriculum? Some people may want to maintain that it shouldn't be changed. But there are at least two kinds of things I'd like to emphasize in thinking about changing it.

One is along psychological dimensions--the kind of work that some psychologists have done--that children from the kind of background we have been talking about need a much more physical, concrete sort of presentation of ideas, and a less verbal one. A re-examination of the curriculum from grade one, from this standpoint, needs doing.

The second kind of thing is the actual turning up-side-down or elimination of topics in the curriculum, not in the direction of the Cambridge Conference, which one way or another is really specified and thought about in terms of the more able children, certainly the top half really. Two topics that occupy much time in the standard elementary school curriculum and that I would mention are long division and fractions. A simply incredible amount of time is spent in teaching long division in grades 4, 5, 6, and undoubtedly one of the reasons for a sense of failure on the part of lower achievers about mathematics and their attitudes toward mathematics as they leave elementary school is the fact that they were not able to master these elementary algorithms. And we're fairly certain from various kinds of test results that they did not master them; furthermore we know that a good many of the teachers have these techniques under very bad control. Now I think we haven't thought very radically about eliminating some of this. For example, if most of the work in long division can be done with a desk computer could we be much more skimpy on teaching long division and put the emphasis elsewhere in terms of how do you get accurate answers using a desk computer? I simply give this as an example. I think it's much harder to give a similar kind of example when we talk about teaching fractions. But the same kind of thinking needs to be done about teaching fractions. I mention these as simply two of the most important topics that are vast stumbling blocks in the elementary school mathematics curriculum for low-achievers.

The second main point I want to mention is that the detailed behavioral analysis of what these low-achiever children are doing with the curriculum and with any new curriculum that is put in front of them is by far more important than it has been in the past. And I hope that any curriculum writing efforts that come out of this kind of conference will not simply say that this is a good thing but build into the curriculum effort itself a detailed and rather complete effort in this direction. We've recently been doing some of this ourselves with forty first graders, and we've found it's been overwhelming. It's not something entered into lightly if you're serious about the details of what the child is doing with the curriculum.

The third point came out in some discussion I had at one of the breaks, and that is the interesting possibility of trying to isolate the intellectual from the non-intellectual problems. I don't mean the practical implementation of the social and political problems, but the problems as they now stand. A number of you here have been involved in working in Africa in the past years and, from the intellectual standpoint, the environment of many of the children in elementary schools in Africa is certainly no better and, in many cases worse, than the environment we're talking about in the inner city or in the rural areas of the South. On the other hand, one has the impression that the set toward education is very much firmer. That is, it's very clearly a better defined goal, a more sought goal, in these African countries than it is for many of the cultures we've been talking about in this country. There is an interesting possibility here of trying to get a deeper understanding of what the intellectual problems are, simply in terms of learning and providing the opportunity to learn versus the problem of overcoming non-intellectual problems in terms of cultural attitudes towards the school and, more particularly, towards mathematics.

B. H. Colvin:

In these seven minutes I've had to prepare, under noisy and confused conditions typical of the socially and culturally disadvantaged, I've been trying to think of some encouraging things to say. Let me begin with a remark which I was tempted to make this morning in connection with Mr. Gregory's comments about mathematics for janitors in Wisconsin. I recall that during my years at the University of Wisconsin we had a most unusual janitor. He had served the Department of Mathematics for many years and, I think, even in the days of Warren Weaver had been used as a very valuable "assistant" in teaching calculus. He had been coached to give the correct answers to such questions as "What is the indefinite integral of x?" and "What is the value of the definite integral $\int_0^1 x \, dx$?" Consequently, he was often in demand in our elementary calculus classes when the students were not doing well. We would call in the janitor and say, "Well, now, Mr. Wilson, can you tell us the value of this integral?" The effects of Mr. Wilson's visit were surprisingly beneficial. The approach illustrates, of course, a "modern" teaching technique which can be exploited in a variety of modifications. At any rate, I felt it might be of interest to Mr. Gregory to supplement his report on mathematics courses for janitors with this story of the successful use of janitors as shills in the classroom!

To continue for a minute in this same reminiscent vein, the most striking personal recollection I have of Mr. Wilson concerns his retirement party. His career as janitor-pedagogue was well advanced when I came to Wisconsin and he retired while I was still there. Naturally, the Department wanted to recognize his special contributions. So we had a little get-together and I think that most of the professors who had been using his help with the "slow-learners" were a bit surprised to discover that Mr. Wilson was having no retirement problems, as many of the faculty were. Indeed, it developed that he owned several of the larger apartment houses in town and that he drove to the party in his new Cadillac! - I'm not quite sure what this anecdote teaches us about under-achievers in mathematics. But perhaps it does suggest the value of novel teaching approaches and shadows forth the future needs for mathematics in occupations for which mathematics is not a strict requirement.

In my second remark I want to mention one or two things which seem to me a little bit hopeful. Of course, I think we all feel that the over all problem we have had laid before us is truly overwhelming. On the other hand, there are some hopeful aspects. I find myself feeling, very strongly, that many of the difficulties we see here involve questions of motivation, of perspective, of over all attitudes and questions related to an understanding of the role of mathematics in our total civilization. There is a lesson here which we can carry over very well into all our efforts to improve the mathematics curriculum, teacher training and mathematics teaching. My own conviction is that much of our mathematics training of recent years has moved away from the physical world, away from the worlds of business, science and industry upon which our lives and livelihoods are based.

I suspect, also, that families who are trying desparately to eke out a livelihood and to improve their economic lot are more often going to ask "What good is this?" "Why should I learn it?" and "How does it relate to the other things I am studying?" These people do not invariably accept learning for learning's sake, as we sometimes wish everyone did. To me this attitude seems a healthy one and I think it is not solely confined to any particular group - cultural or social or economic.

I feel strongly that any progress we can make in answering such questions will be most worthwhile, and especially so for many of the groups we are discussing here. There are fascinating paths to explore, both in the possible programs we have heard described today and in collaboration with programs of other disciplines. Already, there is encouraging progress to report. We can be heartened by the recent accomplishments of the American Association for the Advancement of Science in its Elementary Science Curriculum work. It will, I feel, contribute to a more satisfactory integration of science and mathematics concepts at an early school age.

Also, it is a happy fact that many of the questions which have arisen in this conference have also arisen and have been considered in the AAAS planning and in their program, with which I have been associated. I find it hopeful to consider the possibility of more effective collaboration with other disciplines in future curriculum improvements, especially those starting at the earliest grades. Indeed, to me it seems absolutely essential if we are to make any real progress. Such efforts to present mathematics in a realistic relationship to other segments of our educational program and to relate it to our present culture are important for all students, not just the under-achievers. For mathematics has, in fact, become inextricably interwoven in every fibre of our culture, although, regrettably, we do not always talk, write, or teach as though we acknowledged this fact.

A third comment concerns those questions of vocabulary, reading ability and verbal facility about which we have spoken so much. Success in understanding mathematics, success in teaching mathematics, success in writing curriculum and text material - all are tied very strongly to these skills of language. Although we recognize this language area as a focal point of difficulty in mathematics we have not found it easy to improve the situation, as you know. We must, somehow, eradicate the misconception that mathematics deals with magical symbols and with mystical thought processes which bear no relation to thoughts expressed in words. The point I want to make is that although we are making some progress, as for example in the SMSG material and other recent work (both mathematical and non-mathematical), we still have much to do. This is the case even in these programs for the college capable. Oftentimes, in my experience, the failure to realize the full potential of the SMSG material stems in part from the teacher's deliberate avoidance of the kind of verbal discussion which we tried very hard to build into SMSG presentations. You really cannot demonstrate understanding of concepts and ways of mathematical thought by listing only the final numerical answer to a complicated exercise! Of course, this is really not new nor surprising but I simply point out that the problem is with us still, at all levels, and has many ramifications. It is part of our teacher training problem as well. With under-achievers in mathematics and with their teachers, unfortunately, it will be even harder to improve the articulation of mathematical thought.

Finally, since I am one of the two or three people here from non-academic institutions, I should perhaps say something about what we see in a company like Boeing. I feel sure that the changes we see are typical of those in all highly technical industries. Also, they represent, although in the extreme, the evolution taking place at a slower pace in most industries. Let me just note that we are a sizeable company, with a working population of the order of 100,000 in recent years. Our ratio of technical workers to the total work force is increasing very rapidly. I suspect it is now roughly 1-to-10. Indeed, in many areas of work which are perhaps more representative of future activities the ratio may be as high as 1-to-5. Of course, when we speak of technical people we mean someone who uses mathematics to some degree, since mathematics is the language and the tool of the technical man, with very few exceptions. We will certainly continue to require more and better mathematical training in all facets of our activity. For us today, the point is that many of these activities will involve people from the groups we are talking about educating.

Those of us in industrial mathematics see very clearly that the mathematical requirements are changing. One consequence, of course, is that many workers are being technologically replaced. Their jobs have changed, they must be retrained, and mathematics is often a part of the retraining. We find encouraging cases of those who do well in this retraining, and they are typical of the more hopeful examples described this morning. But many are simply unable to take up the mathematical training needed for the jobs now available to them. Hopefully we can try to do a better job of training the next generation!

But it is not at all a simple task to anticipate the needs ten years hence. Suppose we do ask, say, what will be required in some manufacturing jobs in technical industries ten or twenty years from now. Unhappily, I find it difficult to make really helpful proposals. Let me just suggest the difficulties. I am thinking of these people we are now beginning to educate and who will come up through the school programs we are discussing. They take manufacturing jobs in industry. But the distressing fact is that often you find a manufacturing job which was relatively routine (and certainly non-mathematical) 10 years ago has now taken on a totally different character. The job may have evolved rapidly into a much more sophisticated and complex manufacturing process which borders on very difficult mathematical problems. Perhaps the job required originally the comparative assessment of several alternatives which were easily studied individually and without formal mathematical skill. The modern version of the job may require an optimum choice to be made among alternatives combining 10 or 20 elements in all possible ways. Suddenly there is a need to deal with combinatorial problems involving the assessment of perhaps $10!$ or $20!$ alternative cases. Moreover this must be done very rapidly and perhaps often. If you recall that $10!$ is roughly 3.6×10^6 and that $20!$ is greater than 2.4×10^{18} you can see that our man may need help in enumerating the possibilities! This remains true even with an army of electronic computers to assist.

So you see one of the discouraging aspects of our industrial evolution. Problems of very substantial mathematical nature may come to be involved in jobs which were (and in many respects still are) of very routine nature. The recognition of these problems, their description, analysis, and solution require mathematical skills of varying kinds in many people. I believe this gives a slight indication of the difficulties which underlie any attempt to specify now the kind of mathematical training which industries and businesses of the future will require of their employees.

On this sober note I turn the discussion over to John Williams.

J. D. Williams:

When I discovered that you couldn't avoid being on this program by simply admitting that you knew nothing about the subject, I decided that Mr. Begle must be trying to match a least-likely-man with a least-likely-problem. It does seem to me that the question this audience is considering--What can mathematics, or mathematicians, or mathematics educators do for the least able, the least interested, and the least available parts of the population?--is an unlikely question. I believe we have little to offer, directly, to the culturally improverished, the school drop-outs, and the hard-core unemployed.

The question has its roots in the social system, which is necessarily imperfect, but also which is unnecessarily imperfect. Considering what man's capabilities are for shaping his social system, his environment, and for that matter his own physical and intellectual evolution, it is astonishing how little he does about it deliberately. He does a great deal about it, but very little deliberately and consciously. The greater part of our important affairs are dominated almost entirely by chance. For instance, we all know that the girl you marry is one out of perhaps ten girls whose names you happen to know; that's how much thought and search is given to this important affair. Again, if it weren't for World War II, which we did not plan, probably Mr. Greenberg's IBM machines would still be clanking when they compute, rather than hissing or doing whatever it is that electrons do; so we don't always plan progress even in technology. Of course planning is difficult, and the future is in part, intrinsically inscrutable. But part of our sorry performance is due to lack of trying. The scientists, engineers, mathematicians, and so on among us have spent very little energy studying social systems, in whole or in part. They haven't, generally, been working with economists, political scientists, sociologists, and so on. It seems to me that this is an appropriate way for them to contribute to problems of the kind that face this audience. It is compatible with their numbers and with their talents.

Now, I have no doubt that these problems would still be difficult if the professional scholars of the kinds I'm talking about were working on them. However, I'm sure that ideas would be produced from time to time--and generally they would not be considered feasible. But, the question of feasibility changes with time, and with mood--it depends on how badly you want a solution. For instance, consider the problem of teaching mathematics to a street corner gang in Harlem. Well, perhaps you would get some takers if you offered Blue Chip stamps that were useable at the local pool-hall, and, through logical extensions of the principle, one could doubtless get the whole gang to go for mathematics. But this conflicts with various mores of the society, so we would decide not to teach them mathematics after all. As a matter of fact I do not see why we should want to anyway; there are other more important stones to be moved first.

I have one more comment, on a topic that has come up off and on throughout the day. I'm not an educator by trade, but I've always had a predilection for "reading, writing, and 'rithmetic" as a prescription for educating children. I think that we tend to treat it as a single word, and not to notice the order in which things are stated. I submit that reading comes before writing, and that writing comes before arithmetic. I think this relates to the problems we've been discussing which bear on verbalization, language, and so on. Having seen kids reach college unable to write an English sentence, I, for one, don't care much when mathematics is taught, provided the time is used to teach the language until the kids really have command of it. You don't have to study a lot of mathematics early in life, unless you want to be a professional mathematician; but that's not what we're talking about today.

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Harry Levin

PROJECT LITERACY

The ability to read well is the most fundamental and broadly useful trans-curricular skill. However, despite some twelve thousand published researches during the past half-century the process of learning to read and skilled reading are not well understood. Fruitful hypotheses and findings will come, I believe, only from a fine-grained analysis of the acts of reading.

Four years ago, at Cornell, seven of us - psychologists and linguists - began a basic research program on reading. We held a strong bias as regards to the kind of work which we were equipped to do and which, we thought, was needed. Our bias was the conviction that our research should be basic; that it should stem from a theoretical analysis of the reading process, both psychological and linguistic, and that the studies developing from the analysis should be carried out with the most modern methods of behavioral science. The number of publications bewailing the present status of reading instruction strengthened our faith that a new approach, based on theory and culminating in experimental tests of our hypotheses, was a good choice.

I will briefly describe a few of the findings of our group. The results of a study on variable correspondences indicated that, when subjects are given an initial task of learning a constant compared to a variable list, the latter is more difficult. This problem arose from the condition in English that a letter often has more than one associated sound. On the other hand, the original variable training makes subsequent variable lists easier to acquire. My present hypothesis is that the set for diversity results from training in which the subject must attend to more than one aspect of the stimulus in order to solve the problem or form the association.

A developmental study of the discrimination of letter-like forms showed that discrimination improved from 4 to 8 years of age. What dimensions of difference must a child learn to discriminate for each letter never to be confused with another? Dr. Eleanor Gibson attempted to define what features or dimensions of letter differences actually do serve to distinguish them for the child, or lacking them, made them confusable. The research indicates that there is little confusion among letters except for some obvious instances like O vs. Q.

In another experiment it was found that the proper unit of the reading process is neither the single letter nor the whole word but a higher order invariant derived from grapheme-phoneme correspondences.

Carol Bishop found that letter training has greater transfer value than word training.

In order to discover those rules by which pronunciation can be predicted from spelling, English graphic monosyllables were classed according to the arrangements of letters in them in terms of their pronunciations.

This research must stand on its own as contributions to behavioral science, but we feel that some new directions have been indicated. Now the difficult task of translating these findings to curricula and classroom procedures must be undertaken.

We have a substantial amount of information about the child as he learns to read and about the language that he is learning to read. The child who starts to learn to read has a fantastically elaborate mastery of his spoken language. He is able to produce and to perceive in context all (or almost all) of the significant sound contrasts in the language. This skill includes the disregard of the noise in the phonological system, for example, ignoring the non-functional voice qualities of the speaker (young vs. old, male vs. female, etc.); he can group allophones, respond to dialect variations as being similar, etc. There is probably a difference in his receptive compared to his productive skills. He is able to produce and respond to the morphophonemic variations in the system. He has command over a functional - implicit - grammar. He controls perhaps 5,000 words - though I do not know whether this figure refers to his production or reception. By "controls" I think I mean "understands" in the sense that he behaves appropriately when he uses the word or when it is spoken to him.

We may wonder if the child has the general notion of the relationships between writing and speech. Surely, almost every child has been read to. Does he see this as some adult magic? The book may be an incidental prop to story telling so that the children when they tell stories or repeat from memory have a book open in front of them. In my own experience, children are cued to the story by the pictures and can actually get quite good at phasing the story with the appropriate page. Also, surely, every child has learned to make the correct sound in the presence of orthography: STOP, GO, NAME, etc.

The perception of letter and word forms has often been considered a source of difficulty in reading. I think that too much has been made of this. Gibson's research indicates that there is little confusion among letters except for some obvious instances. A master's thesis under my direction by Gae Edelman indicates that the lowest source of error in grouping word forms as similar is overall outline or shape. The major sources of error, incidentally, are the instances where two words share the initial letter.

There are a variety of other predisposing postures which sometimes are lumped under reading readiness training. Some are specific to reading: going from left to right. Others are general to many learnings, such as listening, attending visually and following instructions.

There are a number of similarities and some important differences between learning mathematics and learning to read. Mathematics has a specified set of concepts which may or may not be presented in various ways to the neophyte. It is true, that some purposes of the new mathematics seems to be to teach the child a process of mathematical reasoning or the use of mathematics for other than specifically mathematical purposes. In reading, we are teaching purely a process which is contentless in itself. Only when this content is used to convey information or for other purposes do we see the consequences of having facility with the process of reading.

I have stated many times that the first part of learning to read is to learn the code, the grapheme to phoneme correspondences. Ultimately, the object is to make access to the code automatic or to make the code transparent so that the purposes of the code become primary. This may be an analog to learning algorithms in arithmetic not for themselves but as automatic devices for problem solving. In reading we have no natural ordering which permits the derivation of the algorithm before its automaticity.

One of the lacks of previous research on reading has been the failure to recognize that many skills are involved. Two broad divisions are evident:

1. Learning of the code. English orthography is an impressive representation of the sounds of the language. A first task is to learn this code.
2. Learning to use the code. This rubric subsumes the primary concern of the educator: reading with meaning.

All that I have said to this point is pre-experimental in the sense that it does not tell us about the process of reading but does give us some notions about the condition of the learner and some notions about the task to be acquired. It is both the actual research that we have done as well as some of our analysis of the reading process that, together with certain national considerations, have given rise to Project Literacy that I now want to describe.

In February, 1964, we organized Project Literacy. This project represents one of the major commitments of the Office of Education to basic research and curriculum development concerning both child and adult literacy. Project Literacy may be thought of as the behavioral scientists' involvement in curriculum revision in the ways that you mathematicians have been doing for the past decade. The major initial effort is to bring together researchers and educators from a variety of disciplines to plan research which, when taken as a whole, will give us more substantial results than any single study can provide. Our intent is to encourage basic research which will then contribute to more applied research. Eventually, we will bring these findings to bear on the actual development and try-out curricula.

We have a very broad definition of relevant research. We believe that much current and potential research in learning psychology, visual perception, cognitive behavior, neurophysiology of vision, child development, descriptive linguistics, psycholinguistics, the sociology of education innovation, research with culturally disadvantaged children and programmed instruction (to cite some examples) are essential to understanding literacy.

Each investigator in our national consortium of researchers will be completely responsible for his own research activities. The project will provide mechanisms whereby the individual scientists can communicate their research strategies, problems and results to each other and when necessary they will be able to meet together.

We will hold four research planning conferences to which scholars, well-known in their fields will be invited. Each of these scholars will be asked to bring with them to the conference, a promising graduate student. Our first conference will be concerned with the psychological bases of reading skills. Such interests as association learning, visual perception, and vision and cognitive processes will be represented. Linguistic and psycholinguistic disciplines and their relationships to reading will be our second conference topic. Those interested in English phonology and grammar and the field of graphophonemics and dialect variations will be included. The third conference will be on motivational and sociological factors in reading. Here we will be concerned with planning research on differences in motivation for learning shown by young children, differences in curriculum as a function of socio-economic status and geographic differences as well as the problems presented by dialect differences and substandard English. The subject of the fourth conference will be curriculum development and techniques of instruction. This will involve people in the planning of curricula in relation to the research problems generated in earlier conferences. A participant in the recent development of mathematics and science curricula will be included.

There are a variety of ways in which Project Literacy could probably work with your group as well as with other curriculum development groups. For one, our interests in solving the problems of literacy have taken us into very broad aspects of child development so that in some ways we can be thought of as investigating a variety of problems that would help us understand the educational process. For example, we have in our consortium, scientists who are concerned with the cognitive development of not only the middle class children with whom we have usually worked but also with under privileged groups. I am confident that if we get trustworthy data on these topics they will be useful not only for understanding literacy but understanding mathematics as well.

Secondly, I have said that reading as such is without content. I think we should consider the adaptation of curricular materials to be used not only for their original purposes but also to teach reading. In this program we might think of the use of some of the early math materials as ways to introduce some facility with the language. Incidentally, I have talked with various people who are developing science curricula about the same problem and I hope that before too long we will be able to try out these notions.

Third point. I have been attracted by two aspects of the mathematics curricula which may be useful quite directly in teaching reading. First, I have been very much taken by the attempt to make mathematics reasonable to young children by such devices as the introduction of the pictorial number line or by the use of set and Venn diagrams. I wonder if it would not be sensible as an analog to introduce the concept of languages and writing systems to the child who is learning to read. We might give him some ideas about ideographic, syllabic, and alaphabetic writing systems and have him devise his own system in a rough analog to having the child use number systems of various bases. Also, one way to conceptualize the process of reading and writing is to think of the units in the writing system as one set and the units in the spoken language as another set and have the child look at the correspondences between these two sets. Granted that the sets are complex as are their correspondences, but I wonder if the child who has some sophistication in these elementary aspects of mathematics might not put them to use in learning to read.

Finally, we are dedicated as you are not so much to the comparison of methods as is traditional in educational research but to a fine-grained analysis of the child's process of learning and thinking. We propose and actually have some members of the

Project Literacy group who will devote their activities to techniques of classroom observation and interrogation of children to understand the effects of the curriculum on them. I would not be surprised if the various techniques we develop might not also be useful in the investigation of mathematics as well as other curricula.

Needless to say, I and the other members of Project Literacy hope that we can arrange a fruitful cooperation between your work on mathematics both for the normal learner as well as the under achiever and the research that we are now starting to do and will continue to do for some years on Project Literacy.

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RECOMMENDATIONS OF THE CONFERENCE

Group A. Schools in the Slum Areas of the Great Cities

In our discussion we soon realized that many of the problems of the slum-area schools are also present in other schools. The low achiever has virtually the same problems regardless of where he goes to school. Help is needed for low achievers in all schools.

We gave consideration to two types of program: (a) action programs based upon our knowledge and hunches as to what and how to teach, and (b) research programs on motivation, helping youngsters to change goals and attitudes, how children learn, strategies for teaching, and what mathematics should be taught.

There is both a teacher problem and a student problem in schools in the slum areas. There are many social, economic, and psychological factors involved, but we confined ourselves to a discussion of the role of the school, specifically in the field of mathematics.

We had a disagreement as to how to begin the development of materials. There was general recognition that motivations, overall background, past experiences, vocabulary and intellectual attitudes of slum area children are far different from those of children for whom past SMSG work has been aimed. Should we start with what we have? Should completely new programs be written? Should present materials be revised, enriched? No consensus was reached, although budget considerations and the success of present experimental programs may contribute to the final decision.

Many in the group felt that motivation should be written into new material for students. Others felt that if we expand, enrich, and make more flexible the mathematics material we already have, then motivation could be taken care of in the new material developed for teachers.

Appended to this report is a summary of the numerous recommendations that the individuals in our group submitted for consideration. The following list of recommendations comprises those issues on which we reached a consensus.

I. Specific Recommendations to SMSG

   A. General policy

      1. Great care must be taken that efforts to improve the opportunities in mathematics education for any group of students do not result in a second class curriculum.

      2. In our attack on the problem, priority should be given to a long-range approach that would begin in pre-school or the primary school years.

   B. Action programs directed at students

      1. Curriculum units, based upon research on how students learn, should be prepared for use where needed.

      2. Some materials should be developed now and programs put into action to meet some immediate needs without waiting for the results of research. Appropriate materials and activities should be provided in the kindergarten and first grade to compensate for deficiencies in pre-school experience with quantitative concepts.

   C. Action programs directed at teachers

      1. Strategy materials, also based upon research, should be developed for teachers. Much help is needed for the teacher--not just in-service mathematics education, but specific suggestions on how to reach these youngsters.

   D. Research programs directed at students

      1. Basic research should be conducted to collect behavioral data on the learning of mathematics, beginning in kindergarten. This research would underlie any long-range curriculum plans.

II. Other Recommendations

   A. SMSG should cooperate with all groups, including curriculum groups, in related fields concerned with problems of slum area children.

Appendix

1. There were a number of recommendations for the preparation of very detailed materials for teachers. Many of these also recommended that a wide variety of methods of presenting specific topics be included.
2. There were a number of recommendations that selected classes be chosen in key areas and that presently available materials be used in these classes under the close observation of mathematicians, psychologists, etc.
3. A number of recommendations were for the preparation of individualized materials which would allow students to progress at differential rates and would thus help with the problems of absenteeism and also the wide spread of basic abilities to be expected in slum area classrooms.
4. Closely related recommendations were for the preparation of non-graded texts which might be used, for example, by bright third graders and slow seventh graders simultaneously.
5. There were some recommendations that deficiencies in pre-school experiences be investigated and that appropriate materials and experience be provided in kindergarten or the first grade to make up for these deficiencies. The slum area child should be helped in the transition from his language patterns to the formal language of the school.
6. There were some contradictory recommendations concerning the content of the elementary school program. Some recommended that the content be the same for all students and that the depth at which the material is treated be adjusted to the student's abilities. Others recommended that a basic curriculum be chosen for all students and that students of higher ability be given additional content. Others suggested a differentiated curriculum for all.
7. There were a number of strong recommendations for close cooperation with Project Literacy and other similar projects.

Group B. Segregated Negro Schools

Although the title assigned to this group refers to a particular type of school, it became clear that the problems under discussion are common to a large class of schools of which the segregated Negro school in the South is just an example. Rural schools in Appalachia or the Midwest are other examples.

Discussion centered on four areas. The most serious general problem was the training and retraining of mathematics teachers.

The second problem was the background of the students. Students in segregated schools are not necessarily students with limited abilities or low potential, but they may have limited experiential backgrounds for school learning.

The third area was the availability of appropriate text material for the students in these schools. Material now available probably does not fit the needs of these students.

The last area had to do with cultural deprivation and how it can be overcome.

I. Specific Recommendations to SMSG

   A. General policy

      1. The mathematical concepts for students with limited backgrounds should be the same throughout the K-6 texts as for other students.

      2. Exercise material should include materials appropriate in different geographic regions and economic backgrounds.

      3. Appropriate material on probability and statistics should be introduced at the proper grade level.

   B. Action programs directed at students

      1. K-3 materials should be revised to take account of language deficiency. 4-6 materials should be revised for clarity, keeping in mind children with poor language background. By looking at SMSG with attention to children with poor backgrounds we may improve SMSG for all children.

C. Action programs directed at teachers

1. Teacher's commentaries should be rewritten with attention to the previous training of teachers in deprived areas. The writing team should have some experience, at least observational, in the regions for which they are writing. The commentaries should be tried out intensively with teachers just as texts are tried out with the students. Problem material in both teacher and pupil texts should be drawn where possible from local and familiar situations. Supplementary in-school material should be made available.
2. Colleges in all socially deprived areas should be urged to promote institutes at all levels. It is recommended that these colleges be given aid and advice by SMSG in preparing proposals and planning courses. The courses for the institutes should be mathematics courses taught by the mathematics departments.
3. The teachers should be taught the course that they are to teach to the children, preferably twice: once in an over-all presentation and again in week-by-week sessions as teachers present the material to students.

D. Research programs directed at students

1. The effectiveness of different approaches to new aspects of a subject should be explored. We believe that the intrinsic interest in the subject is the prime motivation but that different approaches to presentation should be investigated.

   Motivation comes by introducing a problem and then presenting the mathematics necessary to do the problem, e.g. game activities might be used to motivate the study of mathematics.

   The development and use of multi-sensory aids should be considered by SMSG.

2. More information should be obtained on how children respond and learn by observing a sample of classes taught by a person preparing the materials. Observation of the class should be by competent psychologists who should also talk with the pupils.

E. Research programs directed at teachers

1. Ways to insure more active and informed participation by teachers in institute classes should be investigated. Even tricky ways need not be ignored: e.g. a confederate could be used to stimulate class discussion.

II. Other Recommendations:

A. The establishment of out-of-school organized centers for children and adults with both in-school and supplementary materials available, should be investigated.

B. Avenues should be explored that would make the teaching profession possible and attractive to more men teachers, with consequent benefit especially to boys.

C. Teacher training colleges should urge the requirement of an academic major and minor program for elementary teachers.

Group C. Mathematics for the Unemployed

This group discussed three kinds of individuals under this general heading: students who have dropped out of school for one reason or another and whose competence in mathematics is very low, potential drop-outs, and the unemployed whose technical training is too narrow and who need training for new jobs.

I. Recommendations to SMSG

A. General policy

1. For the drop-out problem it is recommended that first efforts should be directed toward preparation of materials for potential dropouts. These materials should be used before the students actually leave school. The same materials might also be used in training programs for dropouts.

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2. Similarly, it was agreed that first efforts should be made to improve mathematics texts for use in vocational and technical programs. The same texts would undoubtedly be useful in training programs for the technologically unemployed.

B. Action programs directed at students

1. For the potential or actual dropout, materials should be designed to provide motivation by examining specific problems of interest to such students, and by placing emphasis on general methods of attacking problems.

   a. The intent of this recommendation is that the new material should place emphasis on having the student learn general methods of attacking quantitative and relational problems rather than on finding solutions by making use of recipes.

   b. The specific problems used for purposes of motivation should be based on the student's own experiences, and care should be exercised that traditional work with computational skills is not permitted to stand in the way of the major emphasis to be put on learning general methods of attacking problems.

   c. Illustrative problem areas appropriate for the materials recommended here are:

      (i) poolroom geometry
      (ii) refunds on bottles
      (iii) elementary mechanics, e.g. problems about gears and wheels
      (iv) game-centered problems
      (v) problems involving extraneous data
      (vi) problems posed through prepared kits
      (vii) slide-rule construction and use
      (viii) problems which may be solved by using nomographs
      (ix) machine shop designing problems involving patterns and scaling

(x) probability problems
(xi) problems involving switching circuits
(xii) problems without unique solutions, e.g. optimization and combinatorial problems

2. Films should be considered both for motivational purposes and as a method of showing students how to attack problems.

3. For vocational and technical programs, mathematics texts should be prepared so that the content relates directly to specific trades and occupations. For example, a large variety of algebra texts should be prepared, all with a common core of mathematics but with the motivational material and the applications in each text taken from a single trade or occupation.

4. Handbooks similar to "Mathematics Handbook for Physics and Chemistry" should be produced, but written at a lower level of reading difficulty and containing much graphic and illustrative material.

5. A set of mathematicians should examine in depth the present and probable future mathematical needs of technicians so that the content of mathematics courses for prospective technicians can be specified more appropriately.

C. Action programs directed at teachers

1. After the preparation of the materials recommended above, appropriate inservice training materials for teachers should be prepared. Some of these should be devoted to new mathematical topics found to be necessary for vocational training programs. Others should be concerned with the materials that emphasize problem solving.

Group D. Mathematics Programs for Students of Low Ability

The group in its discussion of mathematics programs for students of low ability considered the problem in terms of the following dimensions:

1. Type of students
2. Where to begin
3. Content
4. Use of modern educational technology: programed materials, television, films, multisensory aids
5. Approaches to teaching: laboratory, deductive, symbolic, through applications
6. Motivation
7. Selection and education of teachers
8. Effects of (a) class size
   (b) cognitive styles
9. Coordination with other groups

From this discussion, the following recommendations emerged:

I. Specific Recommendations to SMSG

A. General policy

1. There was a consensus that three assumptions often made with respect to the pupil of low ability be rejected. These assumptions are:

   a. That the program for the pupil of low ability should be founded on drill.

   b. That the low-ability child should not be required to think.

   c. That any program for low-ability students should involve little or no reading.

2. It is suggested that any materials prepared must assist as well as make demands on a student's ability to read. There is some evidence to support the view that a mathematics program can make a substantial contribution to the general reading ability of the pupil.

3. It is necessary to recognize that teaching mathematics to low-ability pupils at the elementary level is different from the problem at the secondary level. The emphasis in elementary school should be upon development of the experimental background enabling symbolic activity and abstract reasoning. Increased

emphasis in secondary school should be upon motivating the pupil's learning of mathematics.

4. Courses should be similar to the courses for high-ability pupils. One effect would be to reduce the problem of discrepancies in social prestige.

B. Action programs directed at students

   1. Material should be prepared for slow-learning children for grades K-12 in keeping with the general policies outlined. There was no unanimous agreement on the form this should take. The general consensus favored the preparation of many supplementary units which could be organized by the teacher to meet best the needs of the pupils. A variety of material at all levels should be prepared to make a wide appeal to a variety of motivational interests and should be based on experiences of low-ability students. The use of applications, especially from the sciences, is one approach to this problem. It was emphasized that once you leave the field of science, many applications become local. Further, it was agreed that the emphasis should be on the mathematical principles involved in the applications and that these mathematical principles should not be lost in learning the background material as often has been the case.

   2. Some supplementary materials should be prepared which are remedial and others which anticipate a youngster's difficulties with a concept and reinforce this concept before he proceeds.

C. Action programs directed at teachers

   1. It is recognized that teachers are still central in the educational process. Several proposals are suggested:

      a. A series of conferences (institutes) which would deal specifically with the low-ability child. It is important to involve and interest administrators in this problem.

    b. The preparation of study guides which include not only the mathematics to be taught but which also suggest appropriate methods for teaching mathematics to the low-ability child.

    c. A series of television lessons for the children, these programs to be coordinated with a television series for the teacher.

D. Research programs directed at students

  1. It is suggested that research be undertaken to answer the following questions:

    a. It was noted in statement (2) in General Policy that the problem at the elementary level was different from the problem at the secondary level. What are the significant variables that cause these differences?

    b. Performance of pupils in K-3 SMSG may not be readily predictive of later difficulties in mathematics. How can we predict from performance at the primary levels those who will have difficulty with mathematics? Why do problems in learning mathematics seem more frequent at grades 4, 9, and 11?

    c. What motivates children to study mathematics?

    d. Are there different cognitive styles exhibited by students learning mathematics?

  2. Investigate the use of multisensory aids to teaching mathematics. As mathematics is basically concerned with abstraction, the investigation should determine the level when aids are most useful.

  3. Investigate various formats for presenting material to students. Is there a preferred sequence of the difficulty of ideas?

E. Research programs directed at teachers

  1. By a series of small experiments examine the effectiveness of various proposals for the inservice training of teachers. As well as proposals mentioned

earlier, consideration should also be given to programs that:

  a. Include a period of released time during the school day for inservice training, preparation, consultation with specialists, conferences with colleagues or such programs that will add positively to the effective attack on problems associated with students of low ability.

  b. Use the mathematics "missionary" approach, e.g. an approach in which a well-prepared teacher accepts an appointment to a school to develop suitable programs within the school and provide inservice training plus regular demonstration classes to teachers of the school--a bootstrap approach.

  c. Use consultants in various ways.

II. Other Recommendations

  A. Current practices should be surveyed and a clearing house established which would disseminate information about promising practices.

  B. Assistance should be available to help schools which are interested in experimenting with various approaches to the problem of the low-ability child and which require assistance and guidance in conducting an experiment.

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